Lecture 6

CS 6380 Artificial Intelligence

A* Search: Optimality and Heuristic Generation

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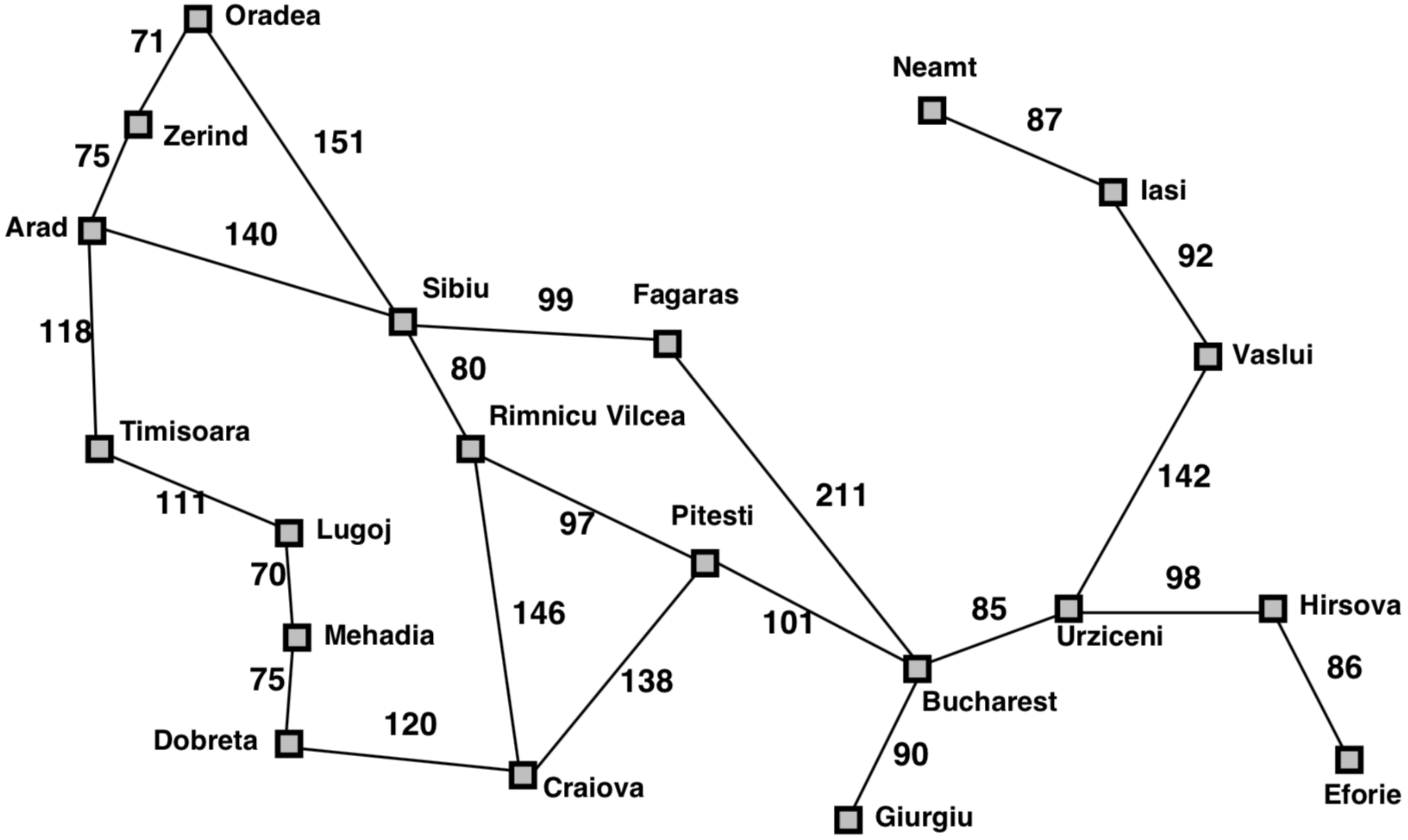
Some material based on lecture slides for CSE 571: Artificial Intelligence by Siddharth Srivastava, ASU.

Logistics and Recap

Heuristic: Example



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy Best First Search

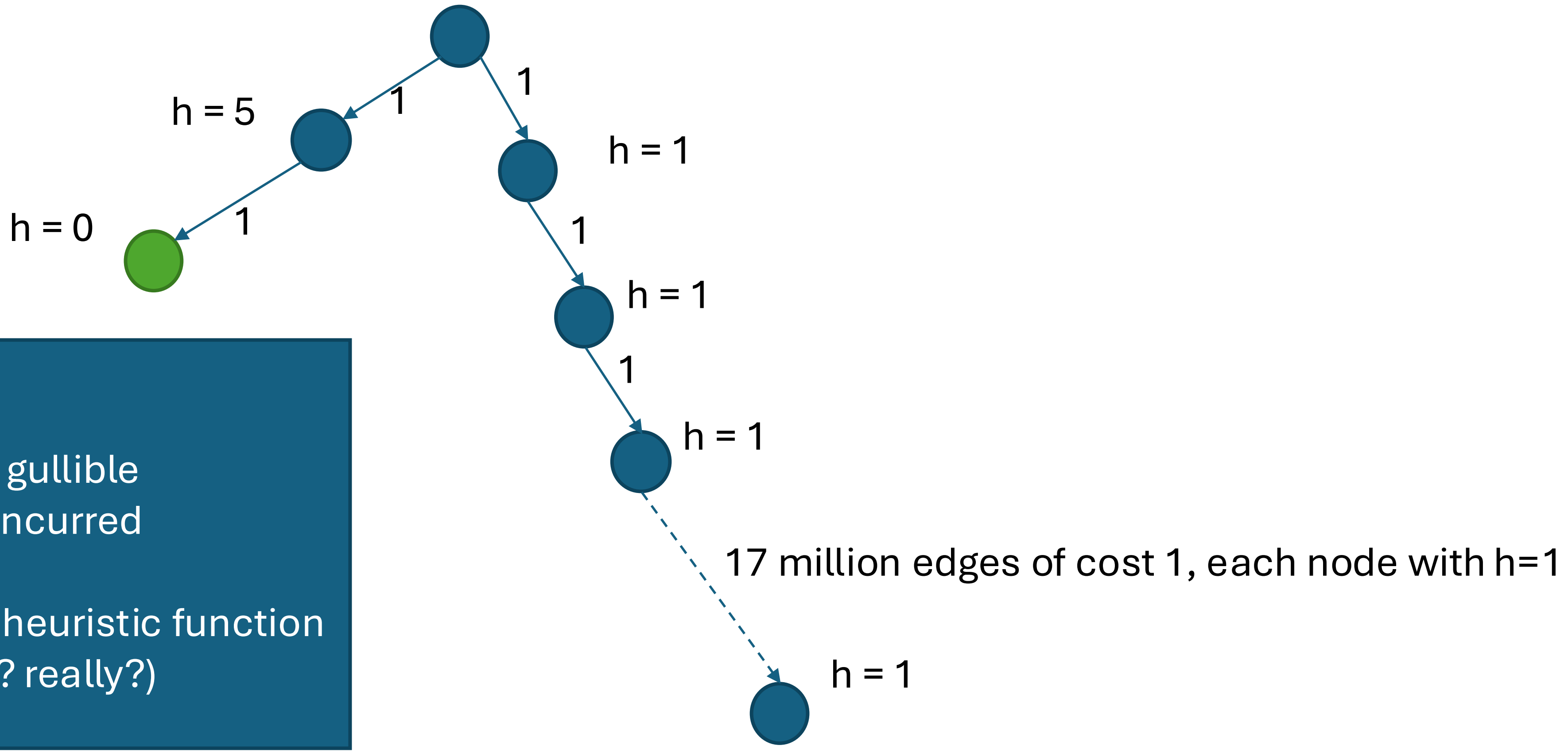
- Implement TreeSearch with following expansion strategy:
  - Expand the node with the least h value

  ↪ appears to be the closest to the goal

Will this be an optimal algorithm?

Greedy Search: Negative Result

- In the worst case, greedy search can expand all other nodes in the graph (even those that are arbitrarily further than the goal) before reaching the goal



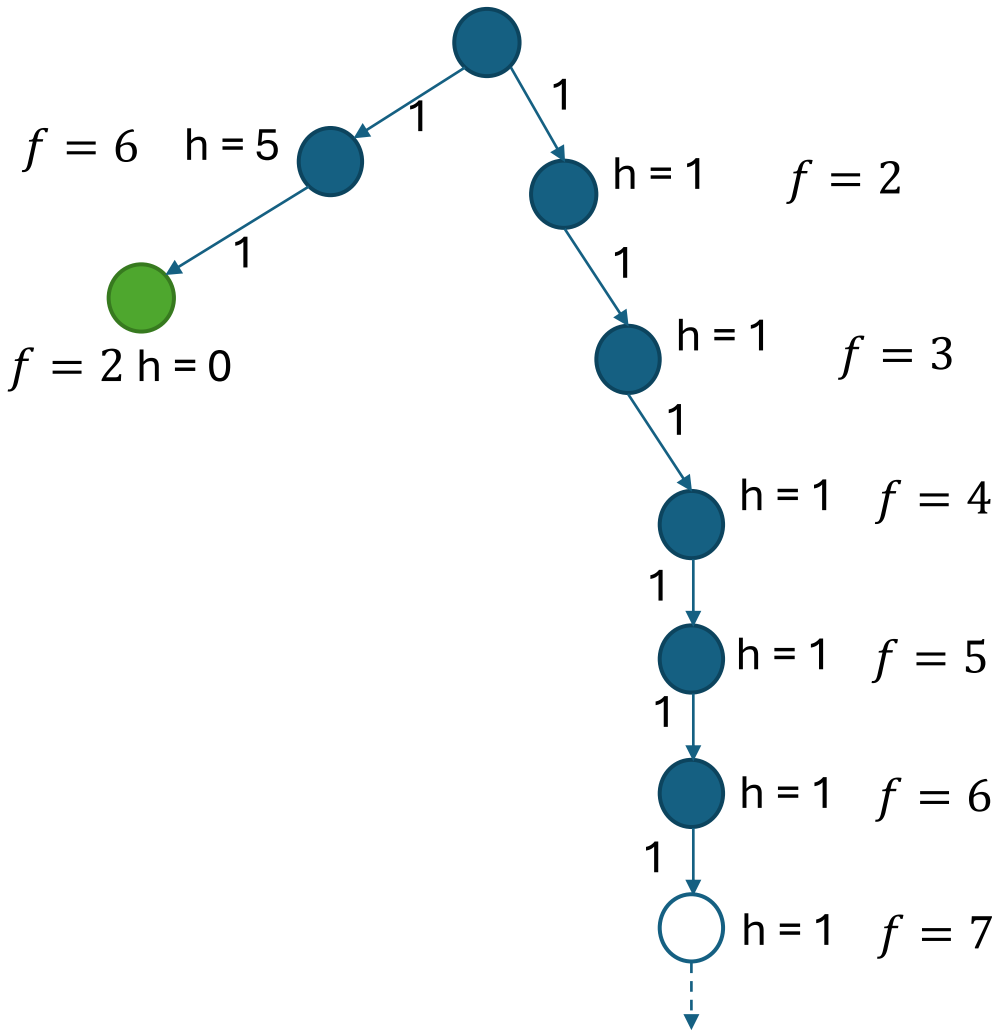
Oops!

- Algorithm could be made less gullible
  - Pay attention to the cost incurred
    - ~ gambler's ruin
- Our stated requirements for a heuristic function were too loose (anything goes? really?)

A* Search

- A* directly addresses those limitations
- $g(n)$ = cost required to get to the node from the start
- $h(n)$ = estimated cost to the goal from n (heuristic, as before)
- $f(n) = g(n) + h(n)$
- Expansion strategy:
  - expand the node with the least f value

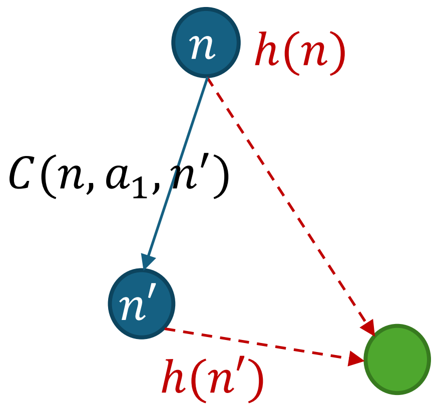
Not immune to bad h
But won't get fooled forever



Optimality of A*

Optimality of A*

- A heuristic is admissible if
  - $h(n) \leq h^*(n)$, where $h^*(n)$ is the optimal cost of reaching the goal from n
  - $h(g) = 0$
  - Admissible heuristics are optimistic about the cost to the goal
- A heuristic is consistent if for every a
  - $h(n) - h(n') \leq C(n, a, n')$
  - Consistent heuristics are optimistic about the cost of each edge
  - This is a more specific constraint



What is the relationship between admissible and consistent heuristics?

Optimality of A*

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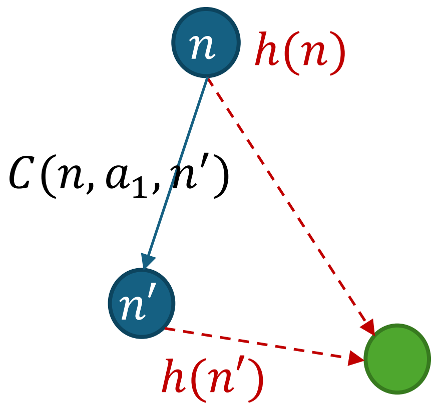


- When using a consistent heuristic, graph search and tree search versions of A* are optimal
- When using an admissible but inconsistent heuristic,
  - tree search version of A* is optimal
  - graph search version is optimal when closed list is managed carefully

Proof Exercise

- To Prove: Let h be a consistent heuristic that is 0 for goal states. Then h is admissible.
- Proof:

Let h be a consistent heuristic.

This implies $h(n) - h(n') \leq C(n, a, n')$, for all a (by definition)… (1)

Let π be a path from $s_0 \xrightarrow{a_1} s_1 \xrightarrow{a_2} s_2 \cdots \xrightarrow{a_m} s_m \xrightarrow{a_g} s_g$, where s_g is a goal node

$C(\pi) = C(s_o, a_1, s_1) + C(s_1, a_2, s_2) + \cdots + C(s_{m-1}, a_m, s_m) + C(s_m, a_g, s_g)$ (by definition of path cost)

$\geq h(s_0) - h(s_1) + h(s_2) - h(s_1) + \cdots + h(s_m) - h(s_{m-1}) + h(s_g) - h(s_m)$ (using 1)

$= h(s_0) + h(s_g)$

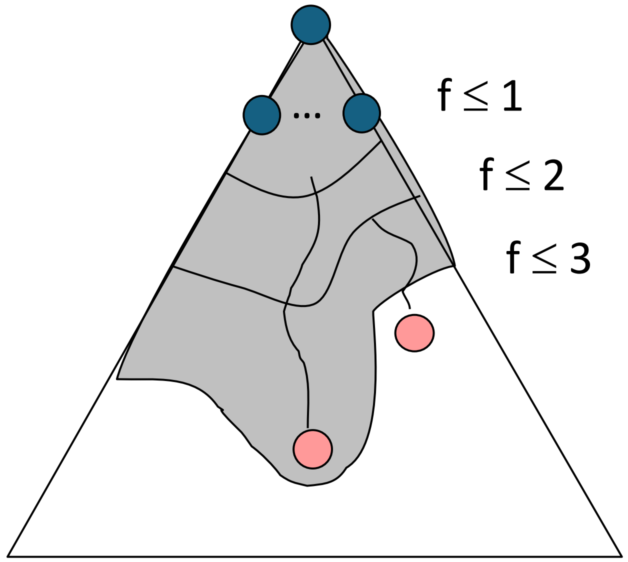
$= h(s_0)$ (because $h(s_g) = 0$, given)

Thus, $h(s_0) \leq C(\pi)$ for any path π from s_0 to goal and in particular for $\pi = \pi^*$, the optimal path from s_0 to a goal

Thus, h is admissible (by definition of admissibility)

Why is A* Optimal for Consistent Heuristics?

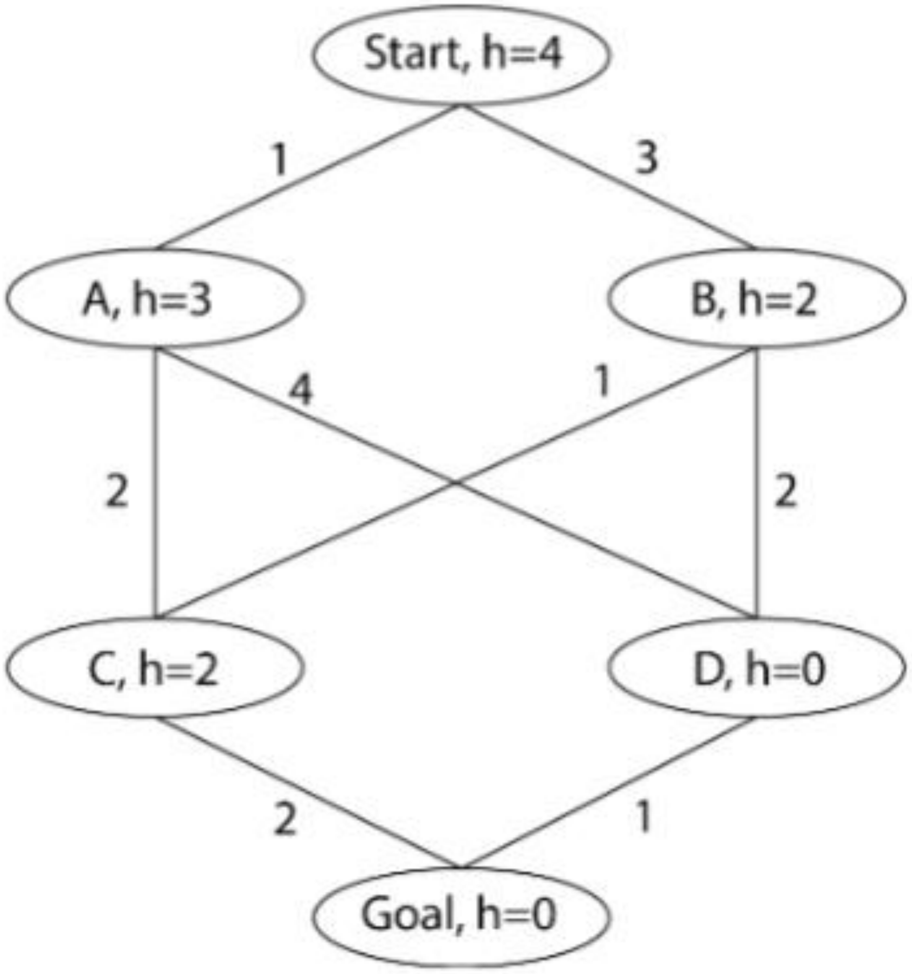
- Lemma 1: If h is consistent and then the f values along any path generated by A* are non-decreasing
  - (proof similar to preceding proof)
- Lemma 2: A* expands nodes in non-decreasing order of their f values
  - By definition of the expansion strategy
- Thus, if A* expands a goal node, by Lemma 2, there cannot be a future expansion that has a lower f value
  - When a goal node is expanded, $f =$ distance to that goal *node*
  - *No future expansion of the goal node will have a lower value of f*
- Q: Will it necessarily expand a goal node if one is reachable?
  - If there are only finitely many nodes n such that $f(n) < C^*$



A* Working Example

Consider the graph on the right. Numbers on the links are **step costs**; the number inside each state is its **heuristic estimate h(n)**. All arcs are undirected.

Start state **S**   Goal state **G**



EVALUATION FUNCTION

$$f(n) = g(n) + h(n)$$

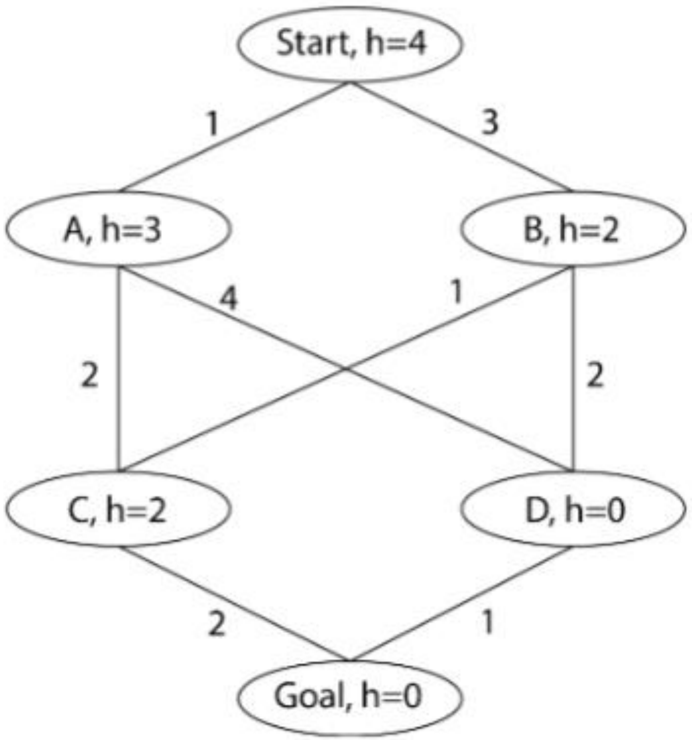
g(n) cost of the best path to n found so far

h(n) estimated cost from n on to the goal

- A* expands the fringe node with the smallest f(n).
- Here h is admissible and consistent, so the path A* returns is optimal.

A* Working Example

Expanded Node	g(n)	h(n)	f(n)	Fringe
S	0	4	4	A(4, via S), B(5, via S)
A (S ->A)	1	3	4	B(5, via S), C(5, via A), D(5, via A)
B (S-> B)	3	2	5	C(5, via A), D(5, via A), D(5, via B), C(6, via B)
C (S->A->C)	3	2	5	D(5, via A), D(5, via B), G(5, via C), ~~C(6, via B)~~
D(S->A->D)	5	0	5	~~D(5, via B)~~, G(5, via C), G(6, via D)
G (S->A->C->G)	5	0	5	~~G(6, via D)~~



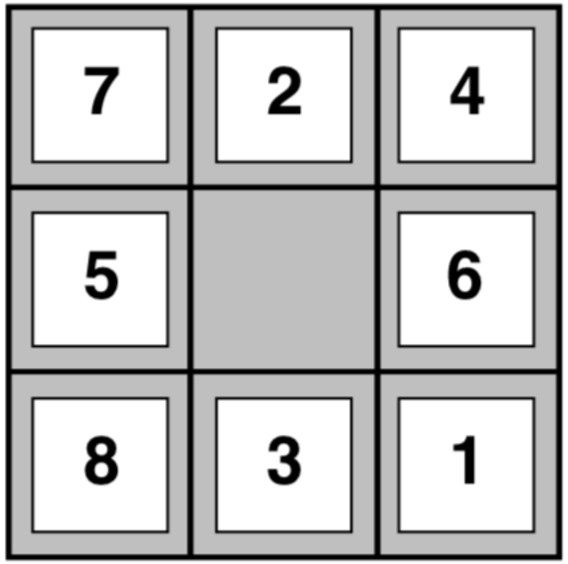
A* Properties

- Completeness: Yes, unless there are infinitely many nodes with $f \leq C^*$
- Time Complexity: Exponential in relative error $\times$ length of solution
- Space Complexity: $O(b^m)$
- Optimal: Yes (conditions on h)
  - Tree Search: h needs to be admissible
  - Graph Search: h needs to be consistent

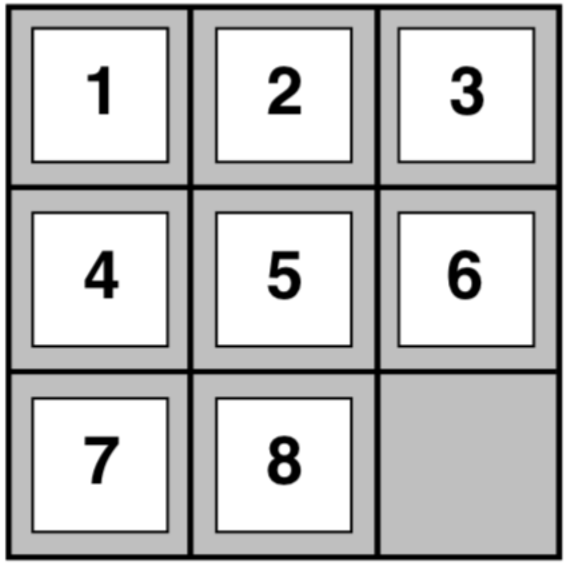
Generating Heuristics

Heuristic Estimates

- Easier in some problems than others!
  - Route finding problem has a natural metric (straight line distance)
  - Most state spaces are not so fortunate
- Online shopping agent
  - State space…
  - Metric?
- Household robot that enters a dark room
  - ”Distance” between a dark room and the same room with the light on

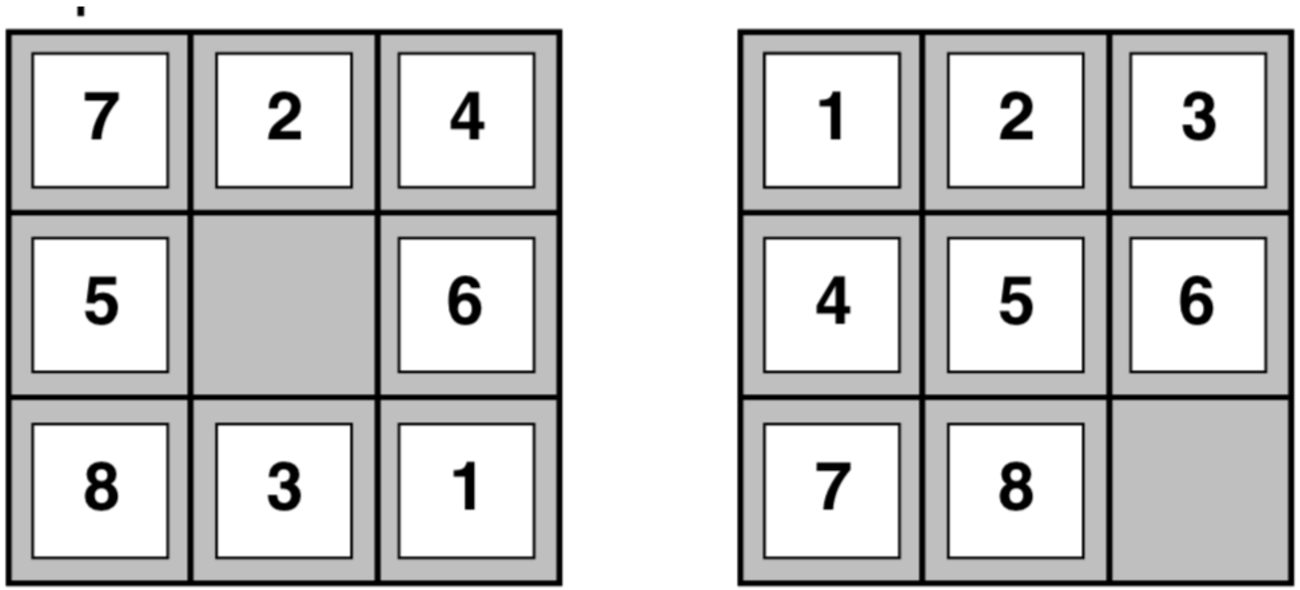


Start State



Goal State

- Can you think of an admissible heuristic?



Start State Goal State

- Can you think of an admissible heuristic?
  - Number of misplaced tiles (6)
  - Total Manhattan distance $(4 + 0 + 3 + 3 + 1 + 0 + 2 + 1 = 14)$

Methodology for Generating Heuristics

- Create a relaxed version of the problem
  - Abstract, or approximate and easier
- Relaxed problems allow shortcuts
  - Optimal solution cost in relaxed $\leq$ optimal solution cost in real
- Optimal solution cost in the relaxed problem constitutes a heuristic
- But, if the relaxed problem is not easy, computing its optimal can be a challenge!

Where A* is Used in Practice…

- Video game route finding problems
- Robot path planning
- Natural Language Processing
- Speech recognition