

Lecture 4

CS 6380 Artificial Intelligence

Uninformed Search

Department of Computer Science and Engineering
Indian Institute of Technology Madras (IITM), India



Logistics and Recap

Resources

- Course Website:
<https://hails-group.github.io/courses/cs6380/jn26.html>
 - Slides posted here
 - Schedule already available
- Announcements: Moodle
- Course Email: pulkityv@cse.iitm.ac.in
 - Subject: [CS6380] <Subject>
 - No URGENT email responses. Please **plan** ahead.
 - We will respond to each email within 48 hours.
 - Any course-related query should be posted on Moodle so that others can also benefit from the answer.



Exams

- 3 Exams:
 1. [Quiz 1] Aug 31
 2. [Quiz 2] Oct 12
 3. [End Sem] Nov 11
- More details closer to exams.

Grading Breakup

- Exams: 70%
 1. Quiz 1 : 20%
 2. Quiz 2 : 20%
 3. End Sem: 30%
- Assignment-Quizzes: 25%
- In-class Quizzes: 5%
- Relative Grading

Announcement

- No class tomorrow (Aug 4)

Advances in Laundry Robots

Task: Fold the clothes.



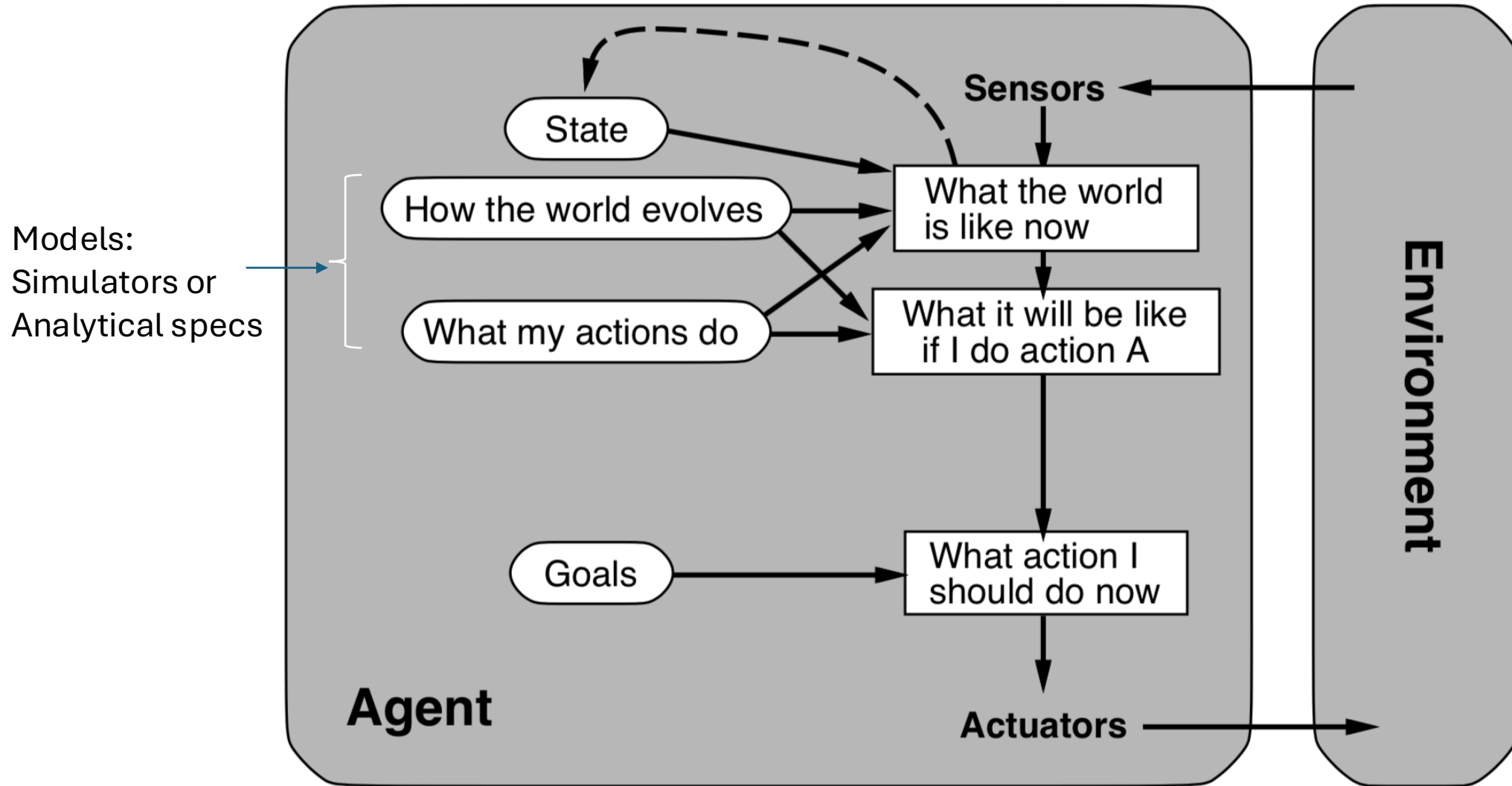
Sync

Ours
(3x action quant)

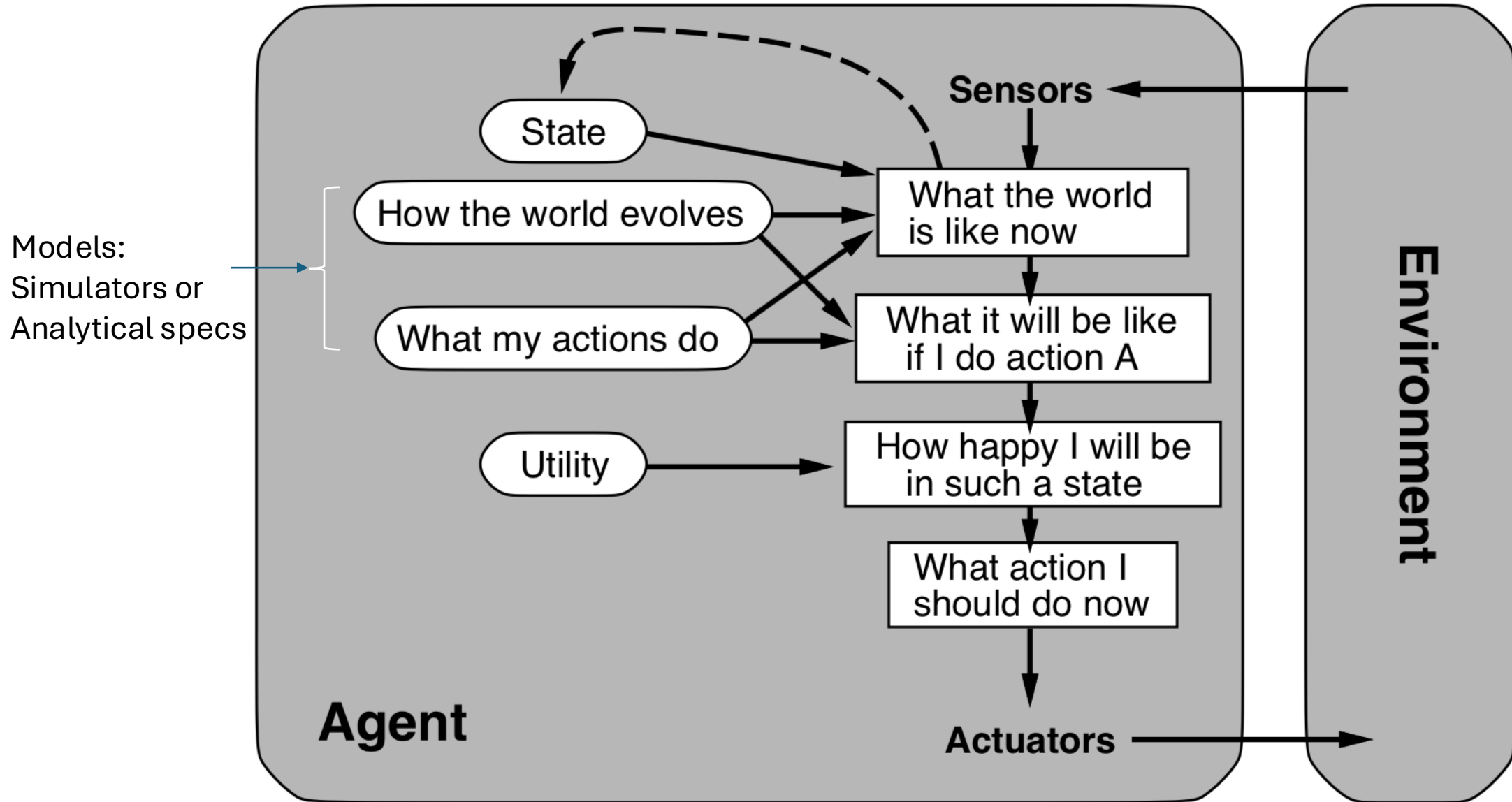
Autonomous, 1x video speed

https://www.youtube.com/watch?v=30G_-bCVvAg

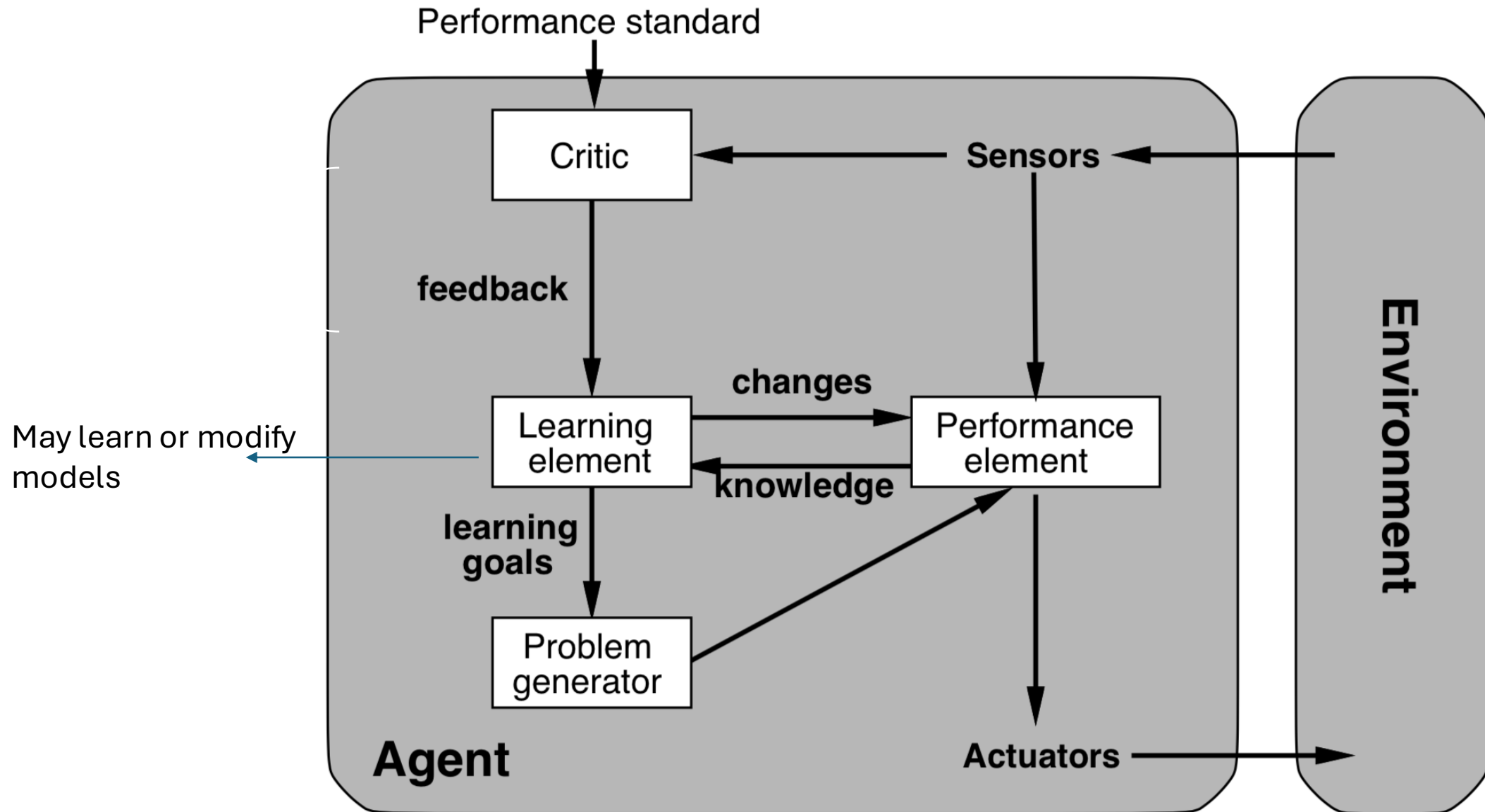
Goal-based Agents



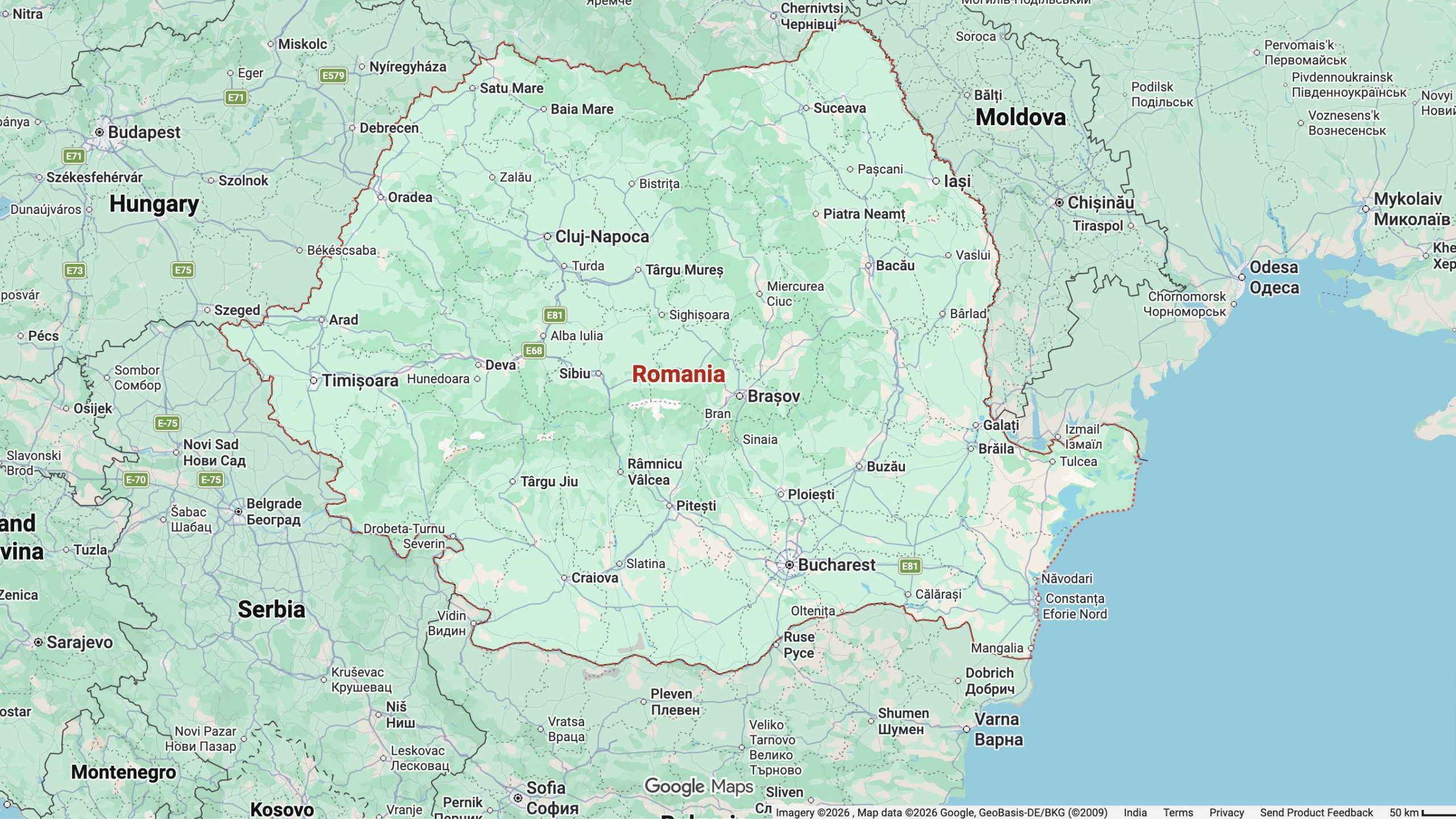
Utility-based Agents



Learning Agents



Problem Formulation



Hungary

Moldova

Romania

Serbia

Montenegro

Kosovo

Sofia
София

Google Maps

The Typical Agent Design Process

1. Represent background information and objective

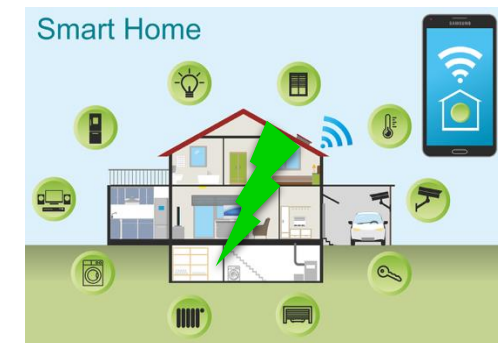
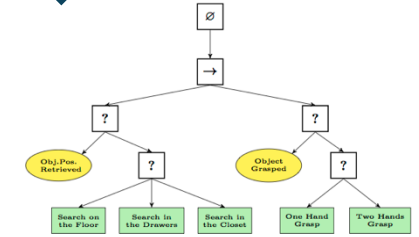
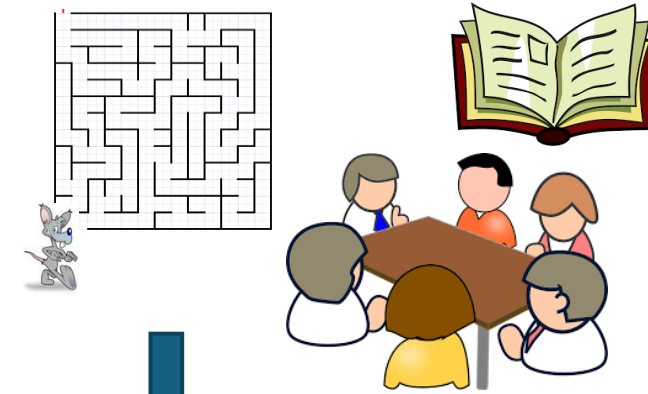
- Multi-step agents require states, actions and their effects on states
- Learned using simulator, modeled using literature/experts, or partially modeled with scope for online learning

2. Compute “behavior” that would allow agent to achieve objective

- Behavior may include contingencies depending on observed system state during execution

3. Execute this behavior

- The bottomline, drives all of the above



How Agents Represent the World

- Every component of an agent must represent the environment somehow. Three levels, in increasing order of expressiveness:
 - **Atomic:** a state is a black box with no internal structure
 - The only operation available is: are these two states identical?
 - Used by search, game-playing, hidden Markov models, Markov decision processes
 - **Factored:** a state is a fixed set of variables, each with a value
 - Two different states can now share some values, and you can say what changed
 - Used by constraint satisfaction, propositional logic, planning, Bayesian networks, most machine learning
 - **Structured:** a state contains objects, each with attributes and relationships to other objects
 - Used by relational databases, first-order logic, first-order probability models, natural language understanding
- **The trade-off:** more expressive representations describe complex worlds far more concisely, but reasoning with them is correspondingly harder.

Representation Drives the Rest of the Course

- **Atomic representations**

- Uninformed and informed search; search in complex environments; adversarial search and games
- Also: Markov decision processes and reinforcement learning

- **Factored representations**

- Constraint satisfaction problems; propositional logic and satisfiability
- Classical planning; probabilistic reasoning and Bayesian networks; most of machine learning

- **Structured representations**

- First-order logic and inference; knowledge representation
- Relational probability models; natural language understanding

- **How to use this when you design an agent**

- Choose the **coarsest** representation that still captures what your performance measure depends on
- Every step up in expressiveness costs you computationally; pay for it only when the problem demands it

The Typical Agent Design Process

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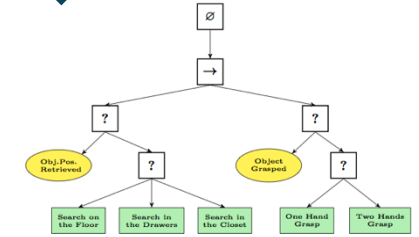
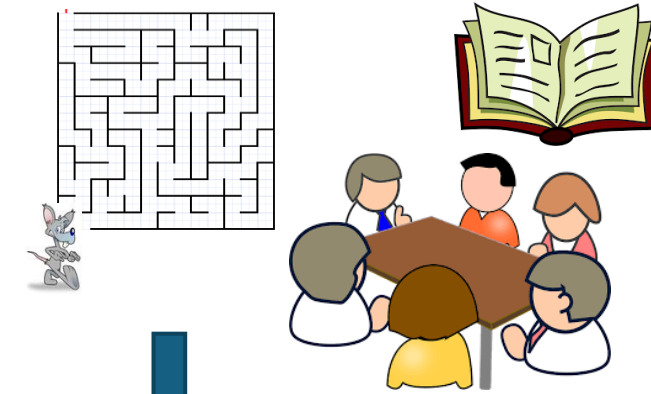
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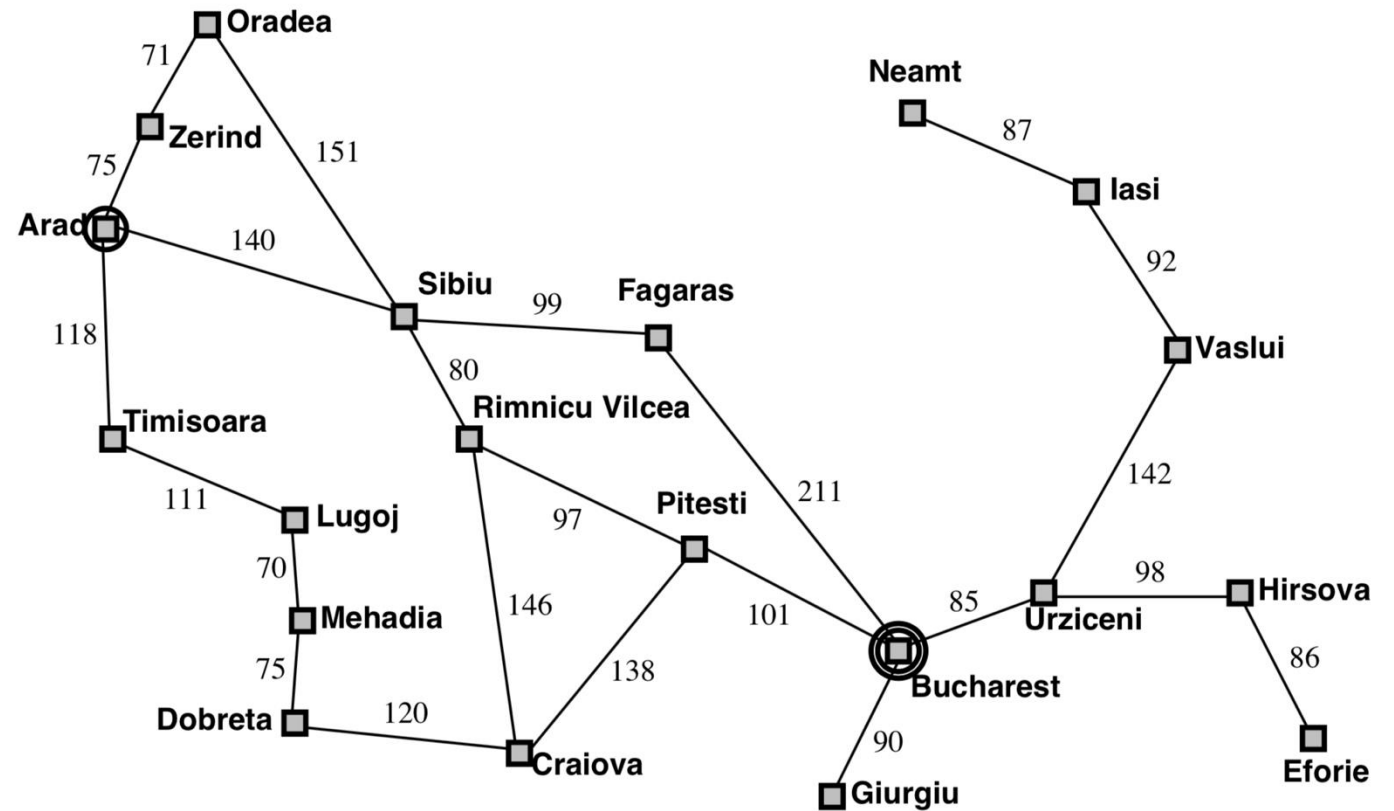
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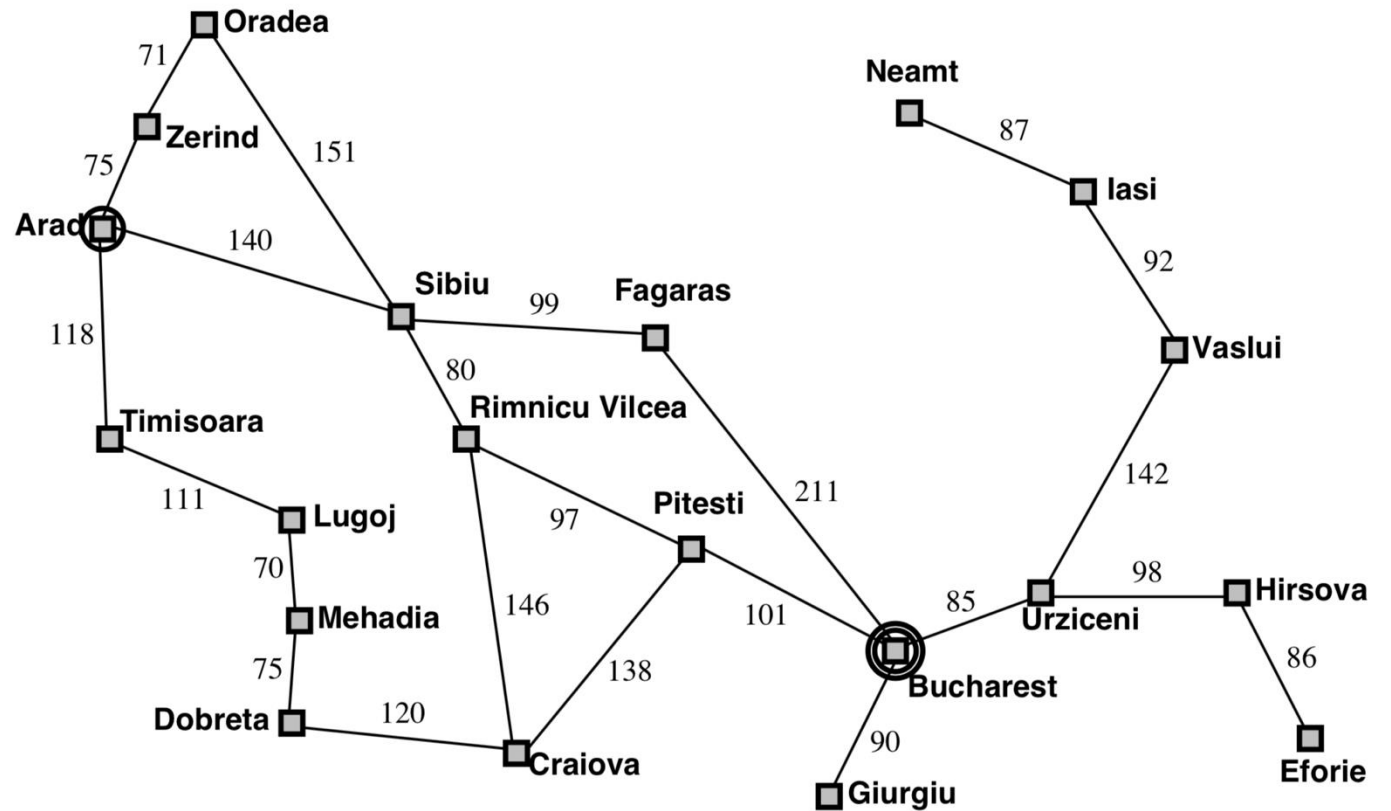
Route-Planning Agent

- State:
 - $\langle \text{current city} \rangle$
- Action:
 - Move along edge
- Performance measure:
 - Cost (length) of path from initial city to goal city



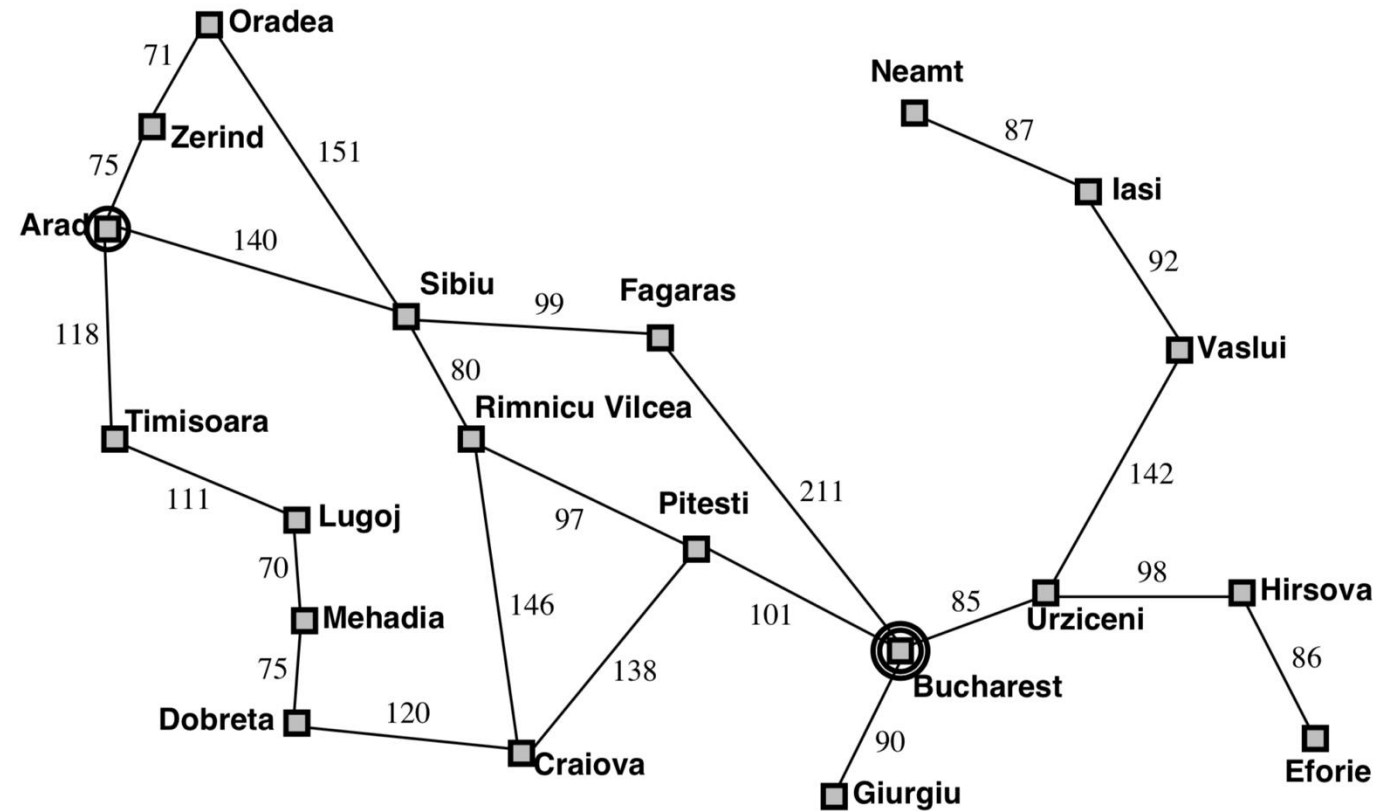
Route-Planning Agent

- State:
 - $\langle \text{current city} \rangle$
- Action:
 - Move along edge
- Performance measure:
 - Cost (length) of path from initial city to goal city
- Such problems are known as *search problems*



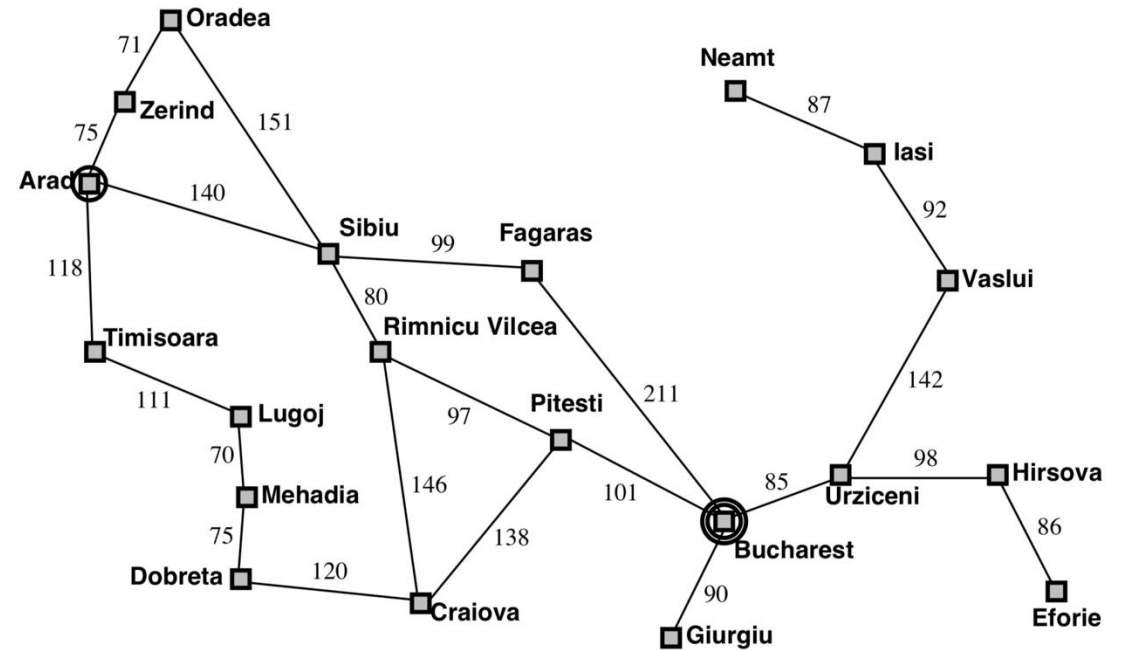
Route-Planning Agent

- Examples of search algorithms?



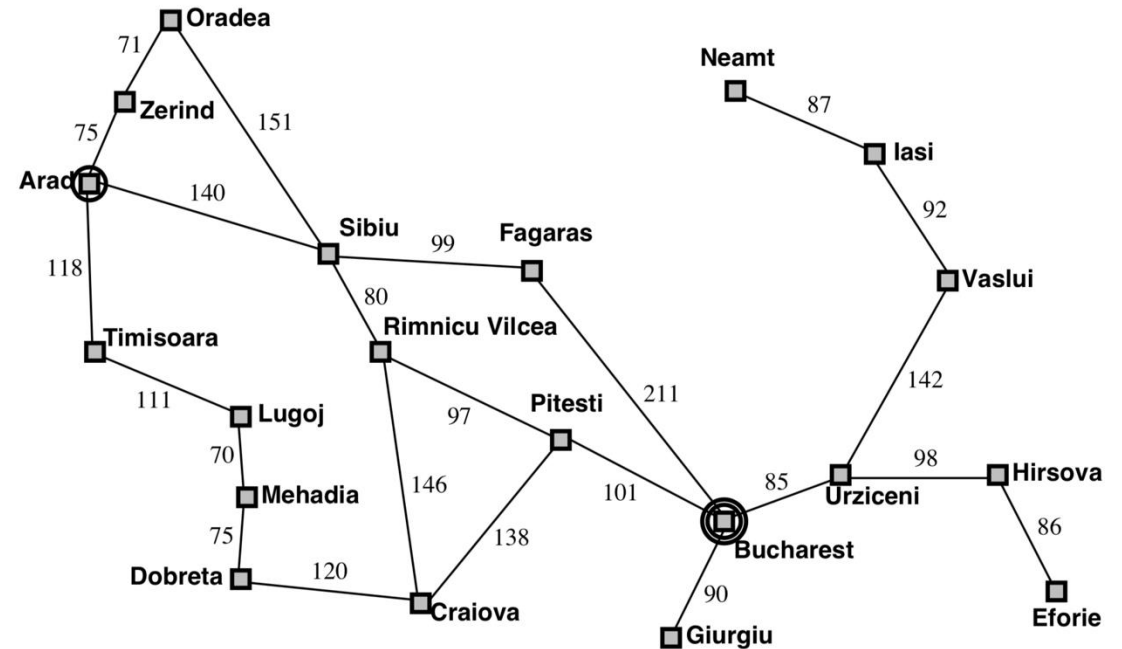
Evaluating Search Algorithms

- Properties of algorithms for computing paths (agent programs):
 - Completeness
 - The algorithm will **find a solution if there exists one**
 - Soundness
 - If the algorithm returns a solution, it will be a valid path in the graph
 - Space complexity
 - Time complexity
 - Optimality
 - Algorithm returns solutions of *minimal* cost



Evaluating Search Algorithms

- Search problems go beyond route planning
- All that's needed:
 - Deterministic actions
 - Fully observable state
 - Initial state, goal state(s)
- What types of problems can be expressed as a graph?



The Typical Agent Design Process

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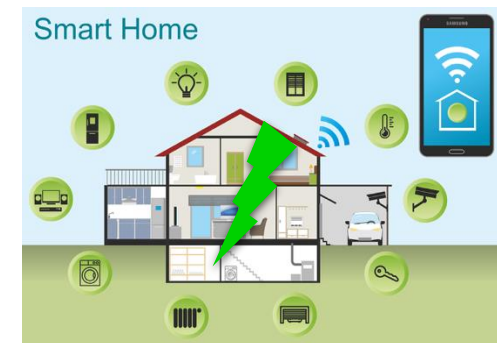
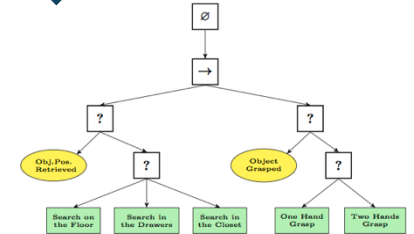
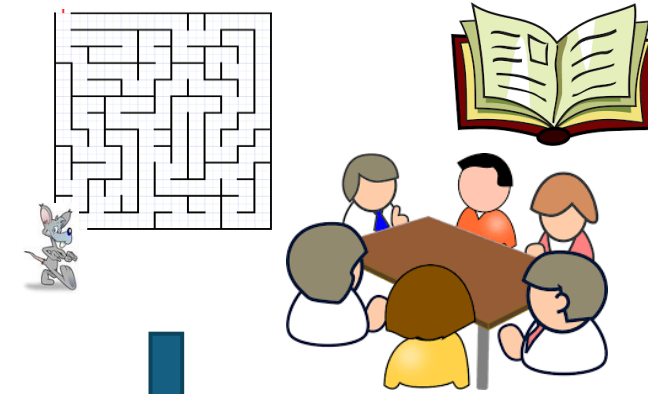
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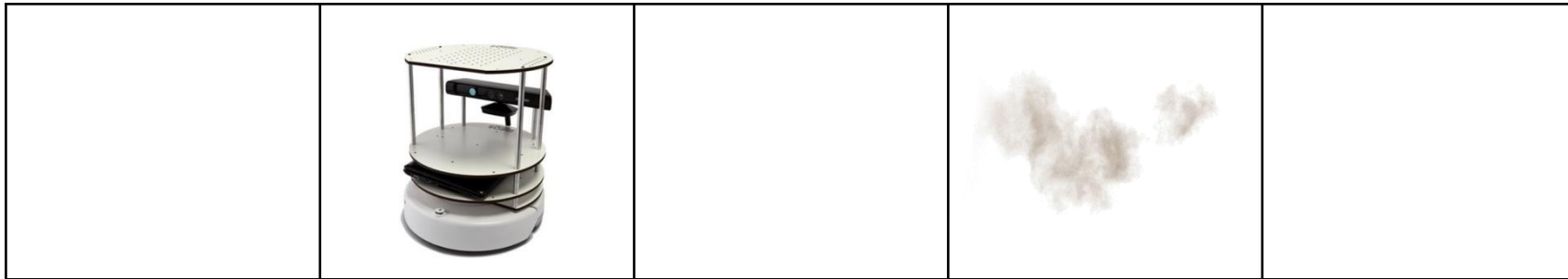
3. Execute this behavior

- The bottomline, drives all of the above



Problem Types and Solution Representations

- Deterministic, fully observable problem
 - $T: S \times A \rightarrow S$; agent observes full state
 - “Classical planning”: solution is a plan, or a sequence of actions



Solution plan: right, right, suck

Forms of Solutions

- Deterministic, fully observable
 - “Classical planning”: solution is a plan, or a sequence of actions
- Deterministic, non-observable
 - “Conformant planning”: solution is a plan, but often no solutions exist



???

Right, Right, Right, Right, Suck, Left, Suck, Left, Suck, Left, Suck, Left, Suck

Forms of Solutions

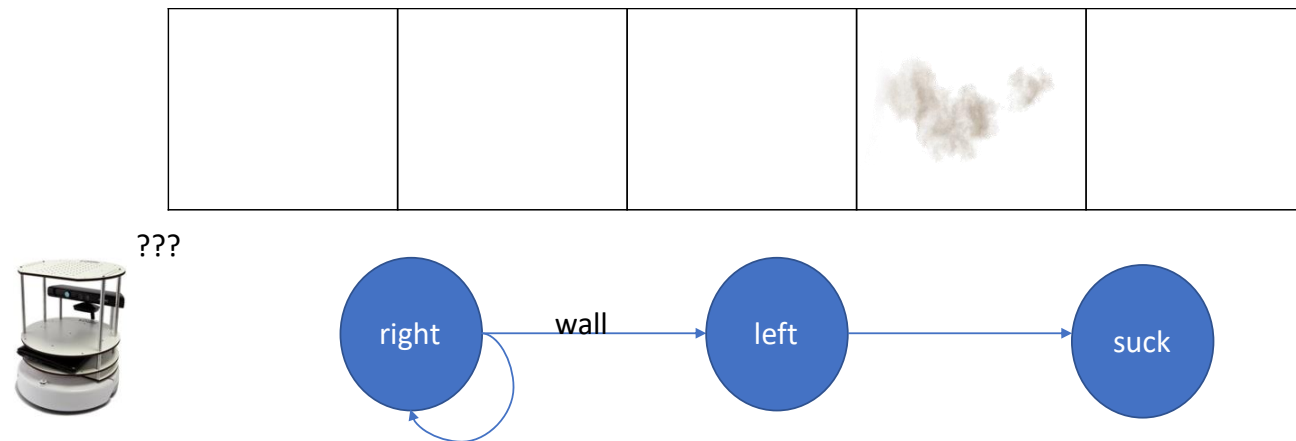
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- Non-deterministic, fully observable:
 - “Contingent planning”: solution can be a policy (a function $S \rightarrow A$)

Non-determinism: a move action may fail



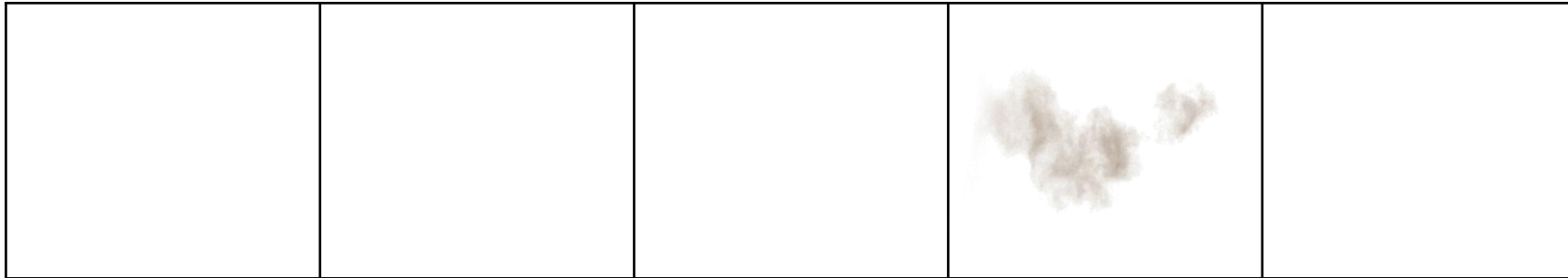
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- Partially observable
 - Solution is a finite state machine

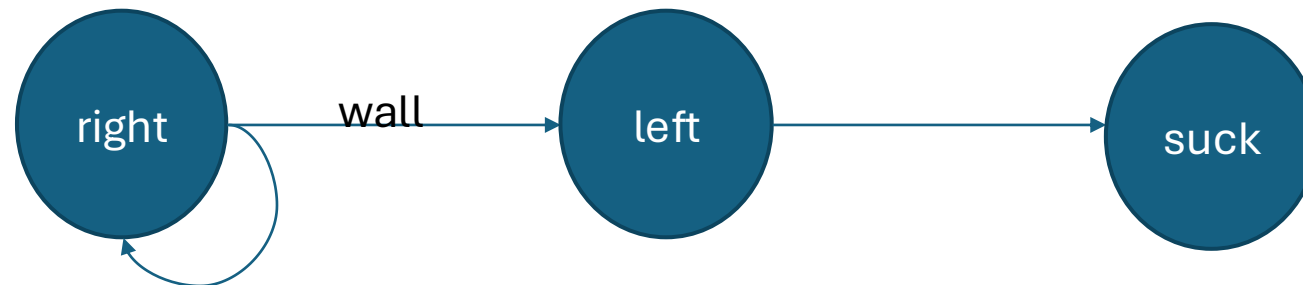


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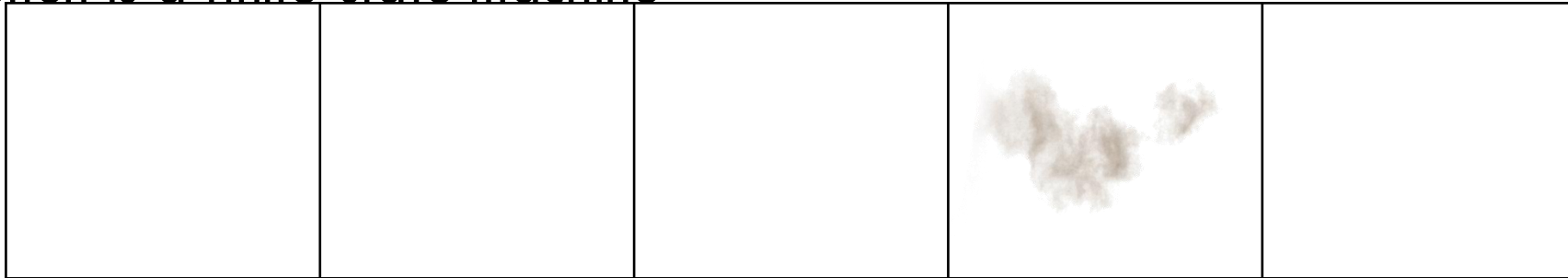


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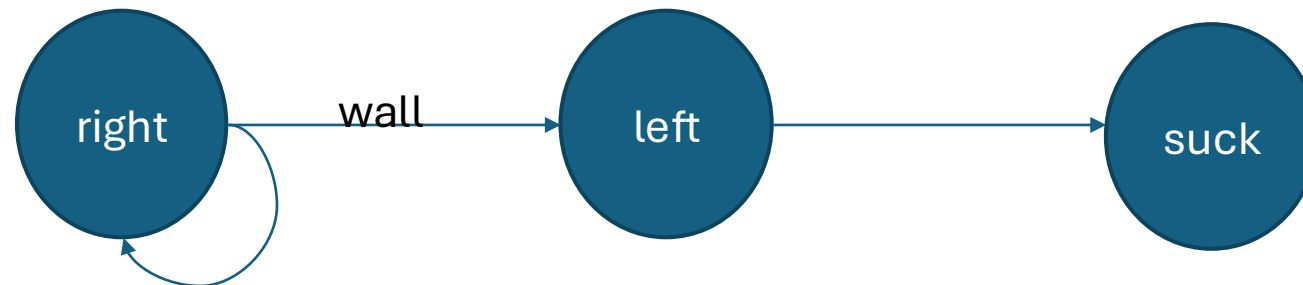


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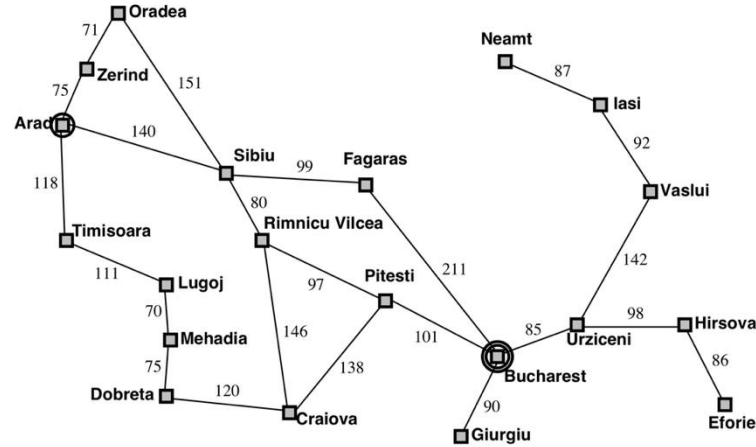
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Deep Dive: Deterministic and Fully Observable Systems

Naïve Formulation of Search Problems

- Given:
 - Graph $\langle V, E, A, \ell \rangle$
 - V set of vertices
 - E set of **directed** edges
 - A set of actions
 - $\ell: E \rightarrow \langle A, \mathbb{R}^+ \rangle$
 - $v_i \in V$ start vertex
 - G goal vertex set
- Find a path from v_i to G



Naïve Formulation of Search Problems

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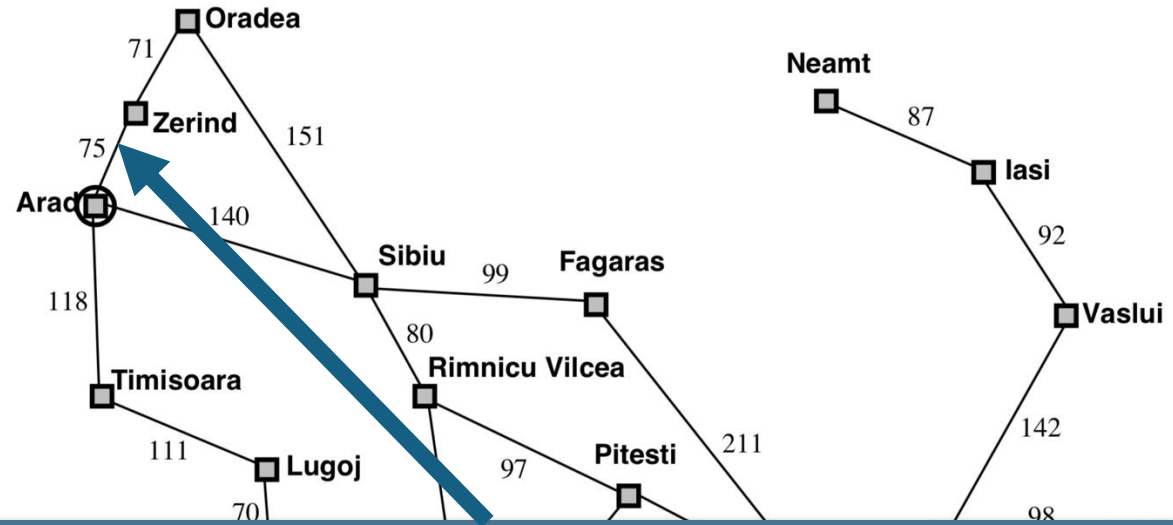
(Arad, Zerind)

(Zerind, Arad)

$\ell((Arad, Zerind)) = \langle drive_Arad_Zerind, 75 \rangle$

- The directed edge from

$\ell((Zerind, Arad)) = \langle drive_Zerind_Arad, 75 \rangle$



Problem with Naïve Formulation of Search Problems

- Given:

- Graph $\langle V, E, A, \ell \rangle$

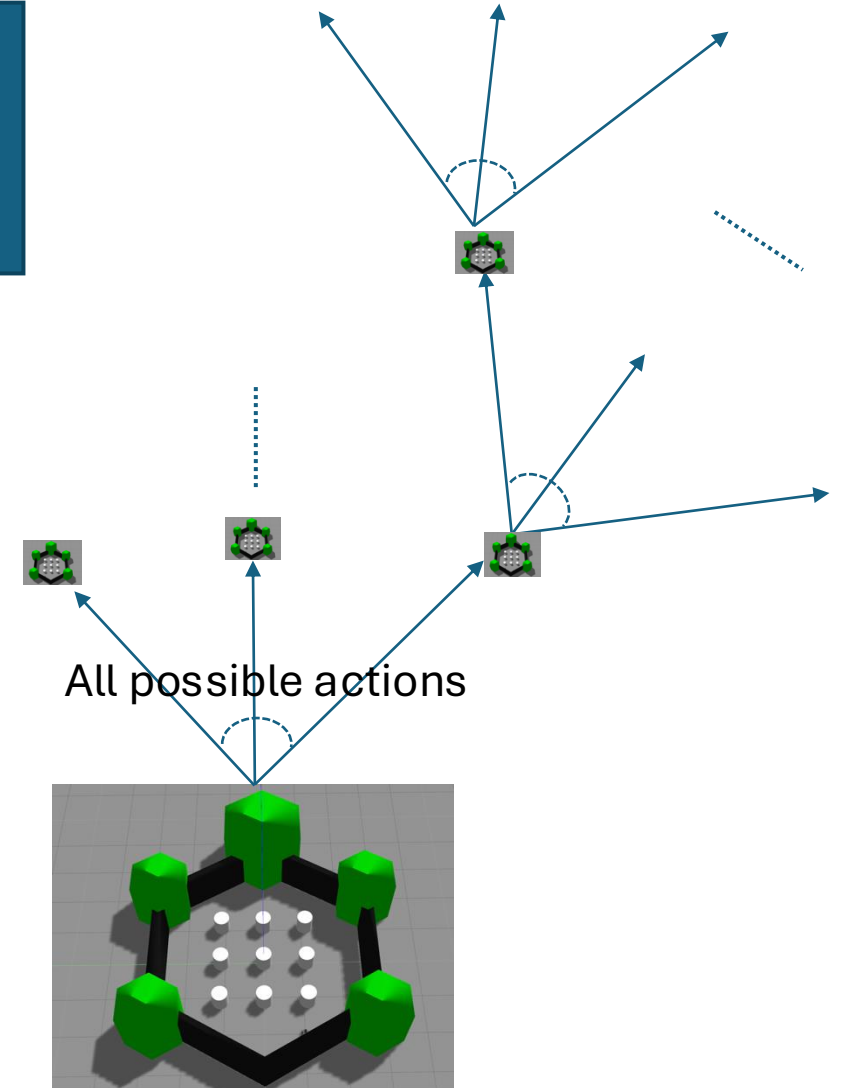
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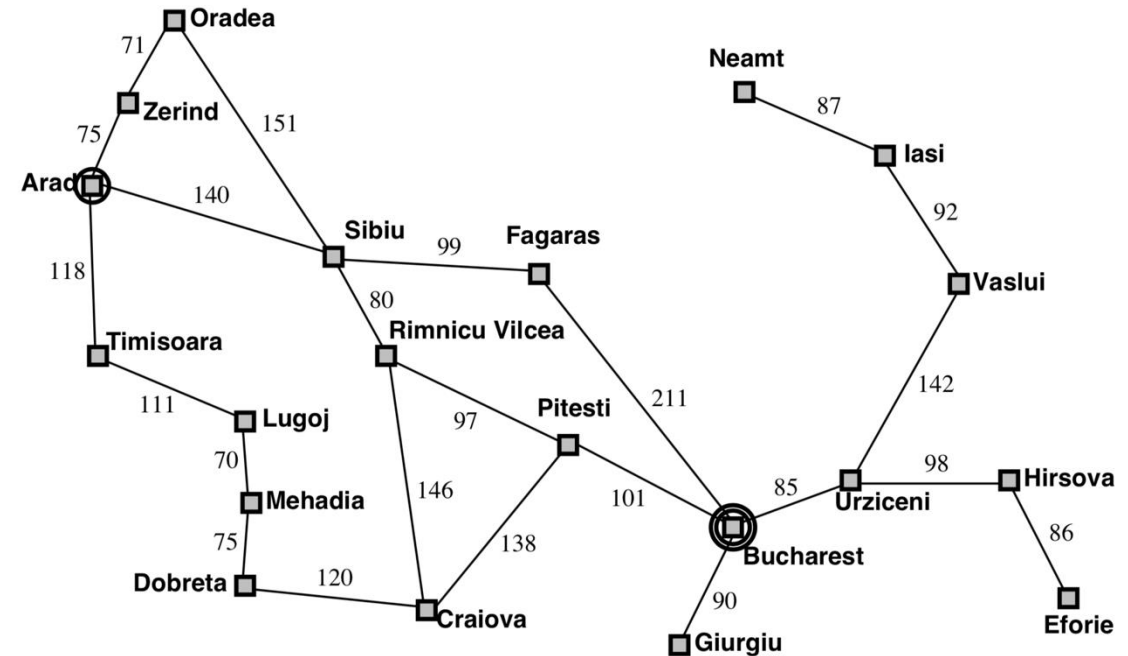
In most cases, the full graph is too large to specify explicitly



Search Problem Defined Over an Implicit Graph

- A **search problem** is defined by
 - S set of possible states
 - Defined, but not enumerated a priori
 - $s_0 \in S$ start state ($\sim$ vertex)
 - A set of actions
 - $N: S \rightarrow S$ successor function
 - $T: S \times A \rightarrow S$ deterministic transition function
 - $N(s) = \{s' : \exists a T(s, a) = s'\}$

Set of all s' that can be reached in one step from s

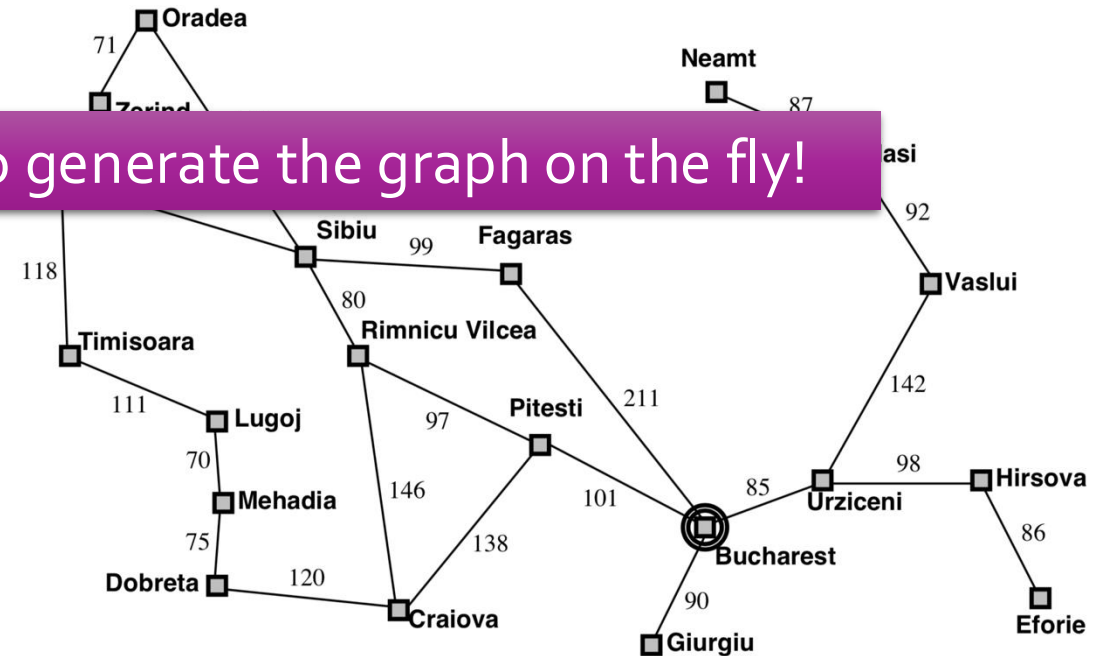


- $N(\text{Arad}) = \{\text{Timisoara}, \text{Sibiu}, \text{Zerind}\}$

Search Problem Defined Over an Implicit Graph

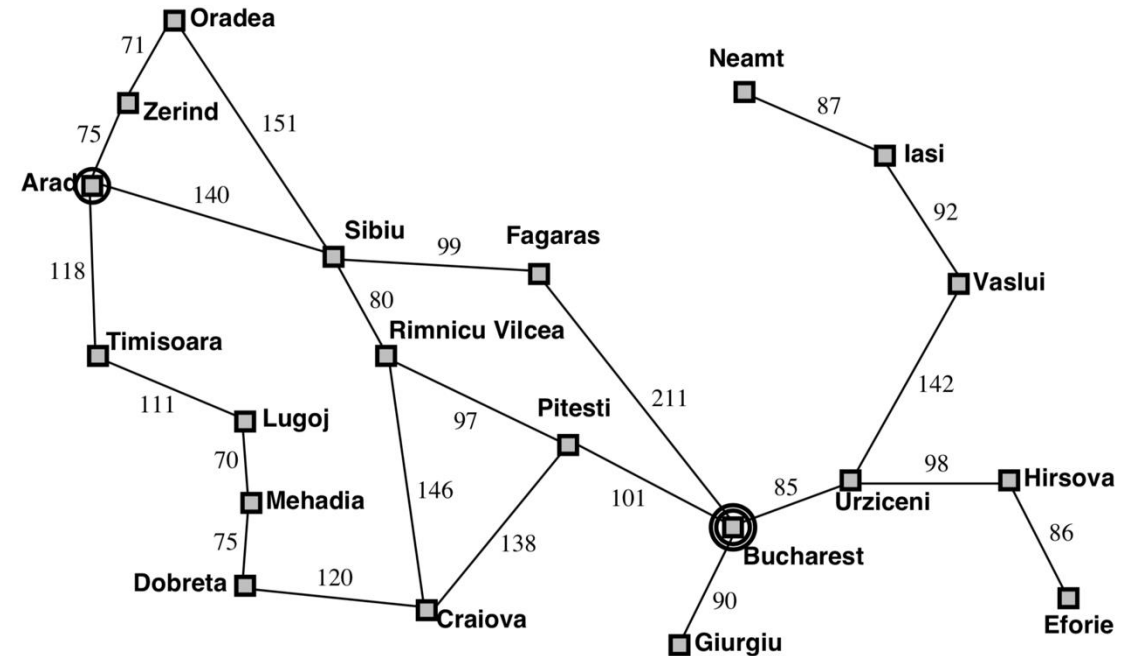
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Allows us to generate the graph on the fly!



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 - $N(s) = \{s' : \exists a T(s, a) = s'\}$
 - $G: S \rightarrow \{True, False\}$ goal test
 - $C: S \times A \times S \rightarrow \mathbb{R}^+$ path cost

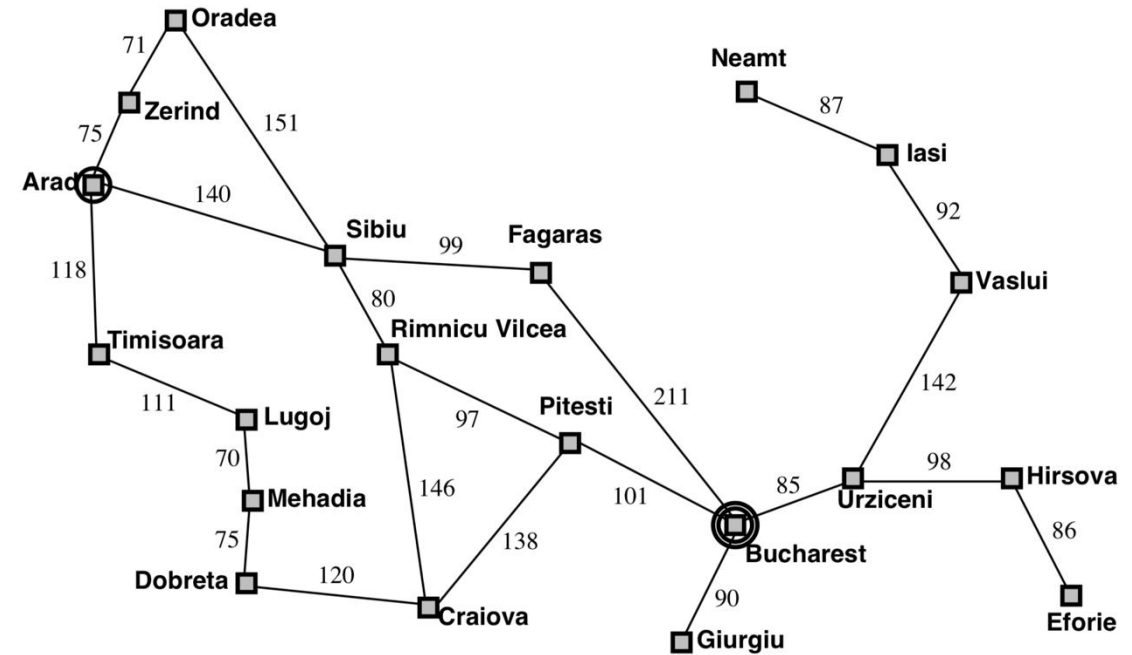


A **solution** to a search problem is a sequence of actions leading from s_0 to a state satisfying the goal test

Role of Abstraction in Modeling Search Problems

- Search problem formulations are models
 - Most models are abstractions
 - Driving from one city to another involves
 - Traffic, one-way streets, fuel considerations, rest stops, ...
- Abstract state = set of real states
- Abstract action = set of procedures

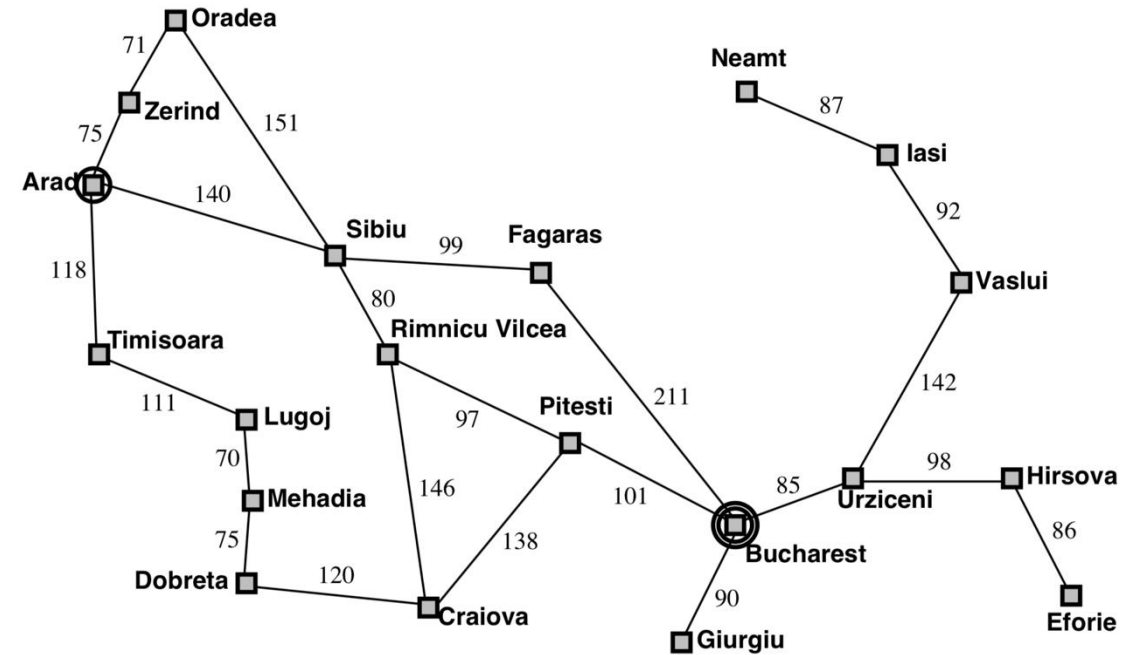
Abstraction is a common and important aspect of modeling



Abstract problem should be easier than the real problem!

Role of Abstraction in Modeling Search Problems

- For the abstract solution to be usable we need:
 - $\forall s, a, s'$ such that $T(s, a) = s'$
 - Every real state in s gets to some real state in s' by following a procedure defined by a
 - “Downward refinability”
 - There should be a way to go from all real locations “in Arad” to some real location “in Zerind”

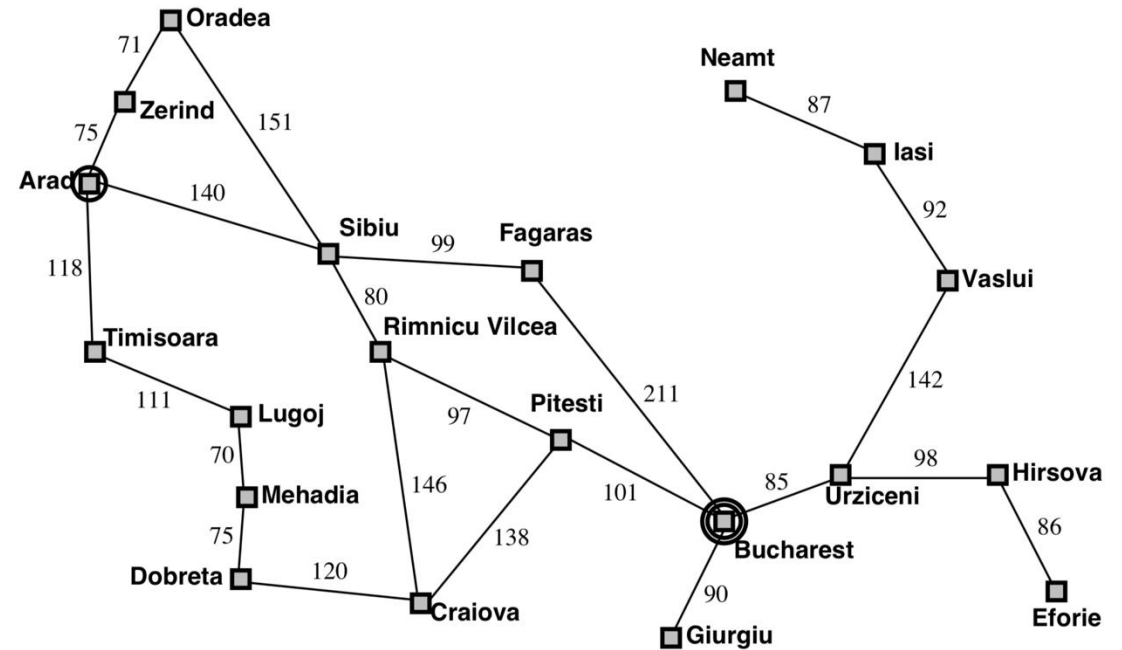


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Solving Search Problems

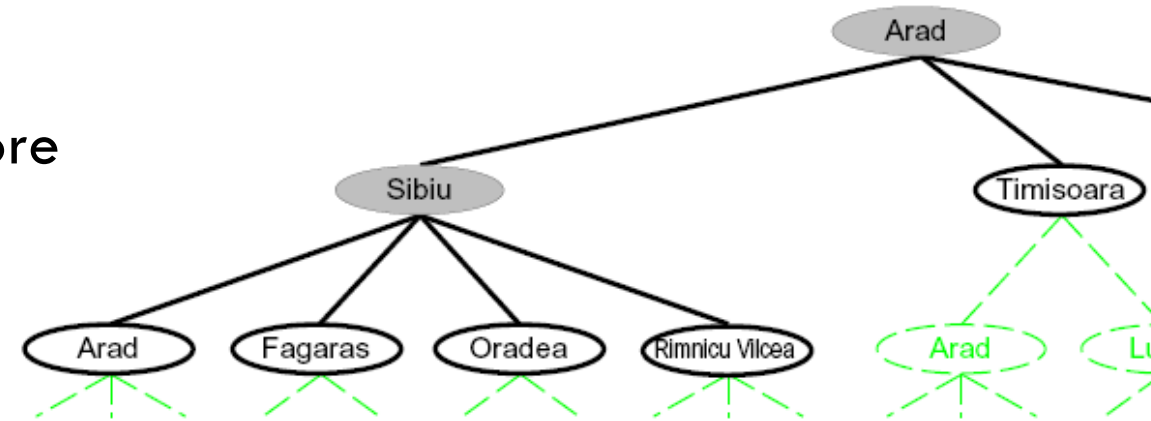
Evaluating Search Algorithms

- Properties of algorithms for computing paths (agent programs):
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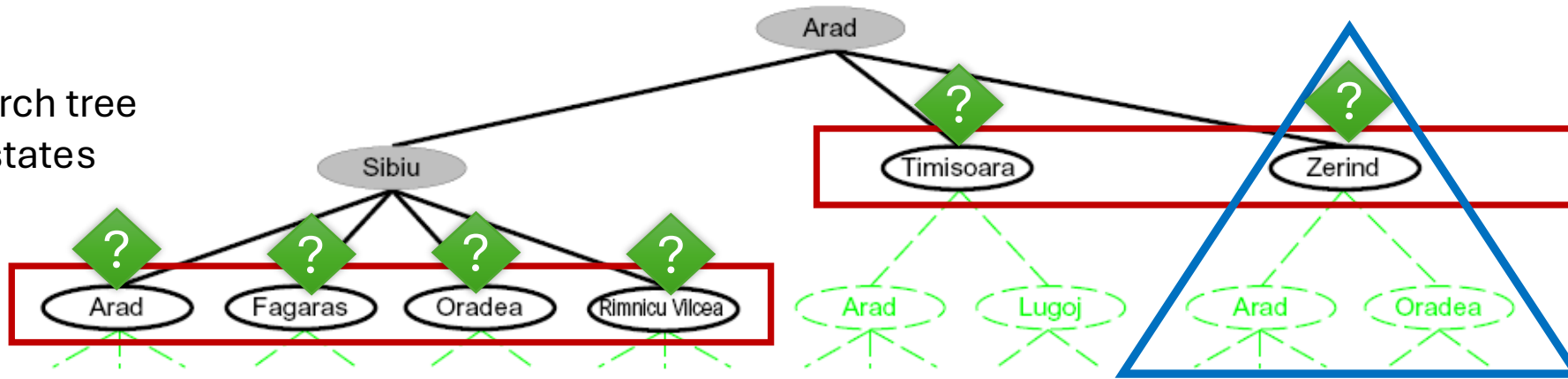
Search Algorithms: Fundamentals

- A **search tree** is used to evaluate possible courses of action in an **offline simulation**
 - Use the successor function to incrementally explore neighbors of explored states
 - “**Expand**” states
- **Node** in the search tree vs **state**
 - A **state** is a **configuration of the agent, environment**
 - A **node** is a **data structure** used in the search-tree
 - **State**, parent, children, depth, incurred path cost $g(x)$
 - Multiple **nodes** may include the **same state** (not desirable)
 - Each **node** corresponds to a **partial plan**
 - The root node includes the start state



General Notions of Search

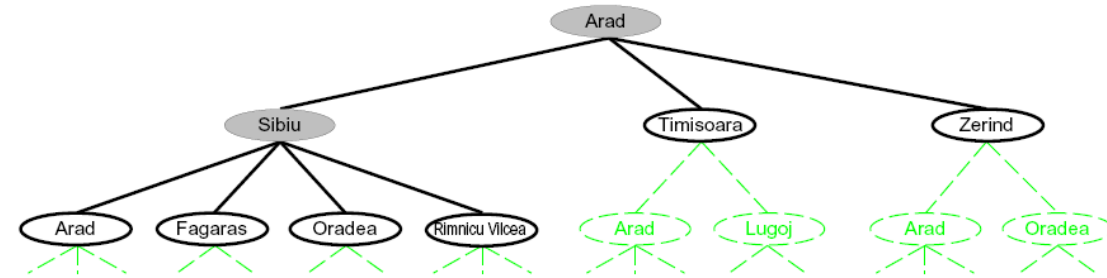
Partial search tree
Nodes vs states



- **Fringe**: partial plans under consideration
- **Exploration strategy**: which node from the fringe should be expanded
- **Node expansion**: generation of successors for a fringe node

General Tree Search

```
function TREE-SEARCH(problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
    if there are no candidates for expansion then return failure
    choose a leaf node for expansion according to strategy
    if the node contains a goal state then return the corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end
```

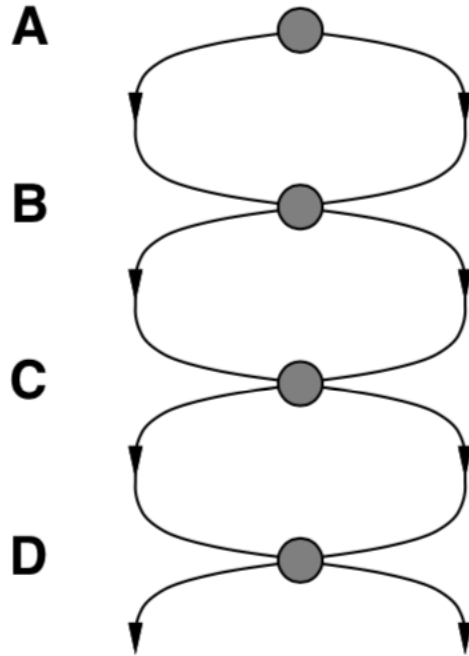


- **Fringe**: partial plans under consideration (set of leaf nodes)
- **Exploration strategy**: which node from the fringe should be expanded
- **Node expansion**: generation of successors for a node

Influences
implementation
of

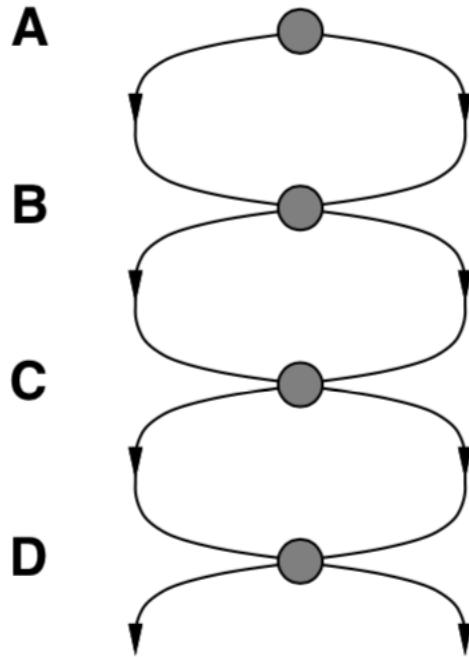
Problem with Tree Search

Doesn't detect repeated states



Problem with Tree Search

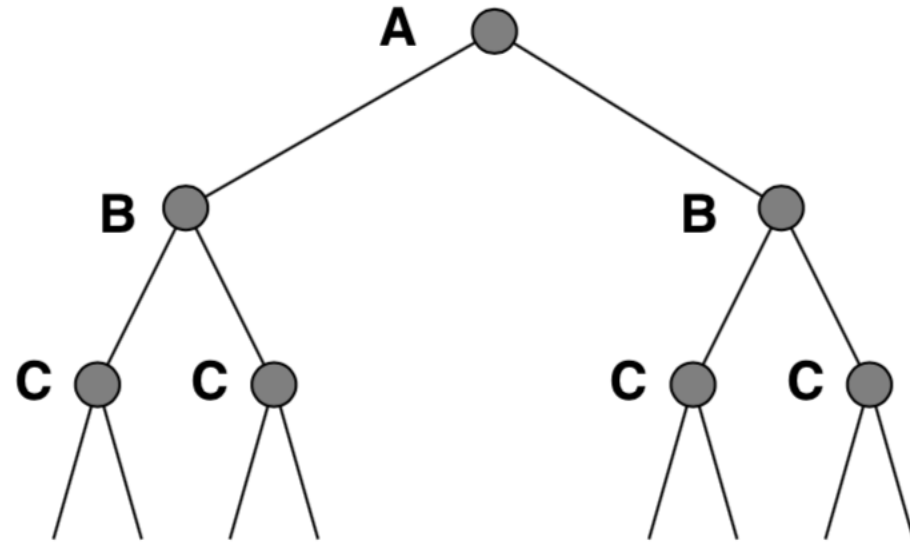
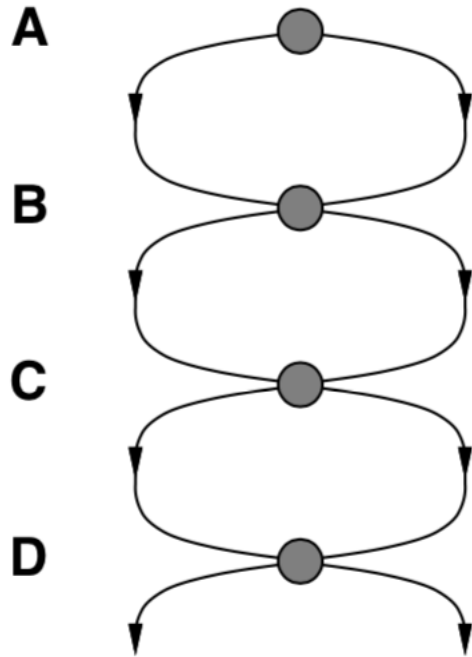
Doesn't detect repeated states



A 

Problem with Tree Search

Doesn't detect repeated states



Easy Fix: Maintain a Closed List

```
function TREE-SEARCH(problem, fringe) returns a solution, or failure
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE(node)) then return node
    fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function GRAPH-SEARCH(problem, fringe) returns a solution, or failure
  closed ← an empty set
  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node ← REMOVE-FRONT(fringe)
    if GOAL-TEST(problem, STATE[node]) then return node
    if STATE[node] is not in closed then
      add STATE[node] to closed
      fringe ← INSERTALL(EXPAND(node, problem), fringe)
  end
```


Easy Fix: Maintain a Closed List

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For clarity in illustrations, we
will use the tree-search

variant

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    if STATE[node] is not in closed then
      add STATE[node] to closed
      fringe ← INSERTALL(EXPAND(node, problem), fringe)
  end
```

Uninformed Search Algorithms

Uninformed Search

- Uninformed, or undirected search algorithms operate without any consideration of how far a state is from a goal
 - Good for situations where distance to a goal cannot be estimated
- We will learn about:
 - Depth first search
 - Breadth first search
 - Iterative deepening search
 - Uniform cost search

Depth-First Search

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    if GOAL-TEST(problem, STATE(node)) then return node
    fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

- **Exploration strategy** (Remove-Front): returns the deepest node
- **Fringe is implemented** as a LIFO stack

Videos of some these algorithms: <https://www.movingai.com/SAS/EXP/>

Organized by Nathan Sturtevant, University of Denver

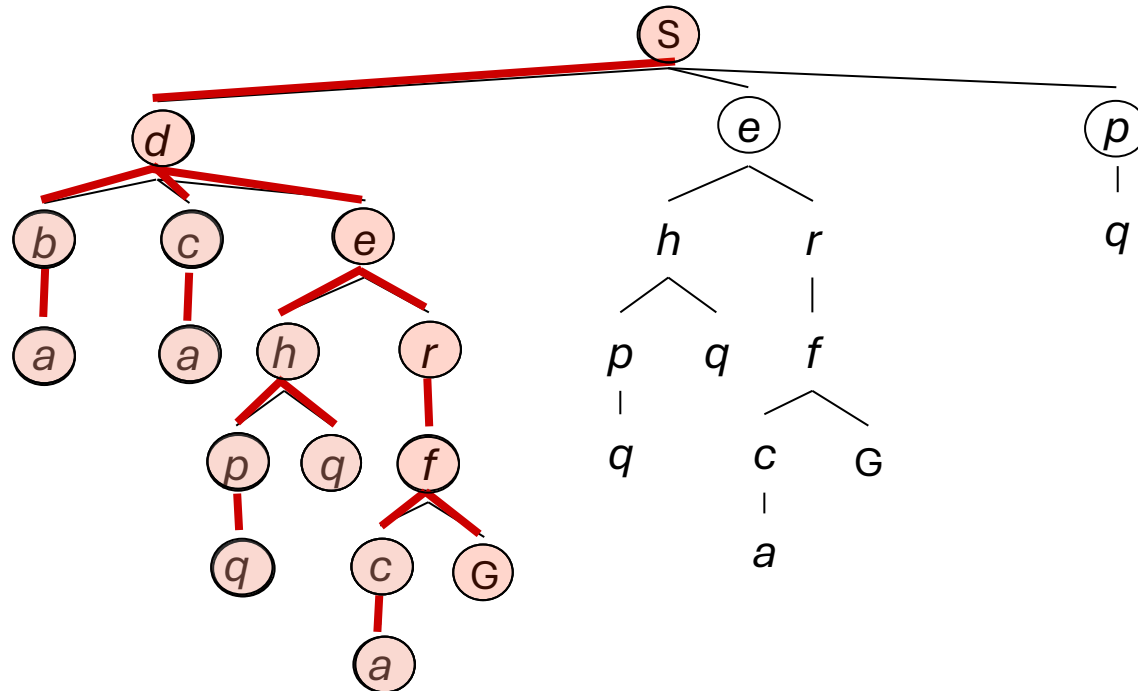
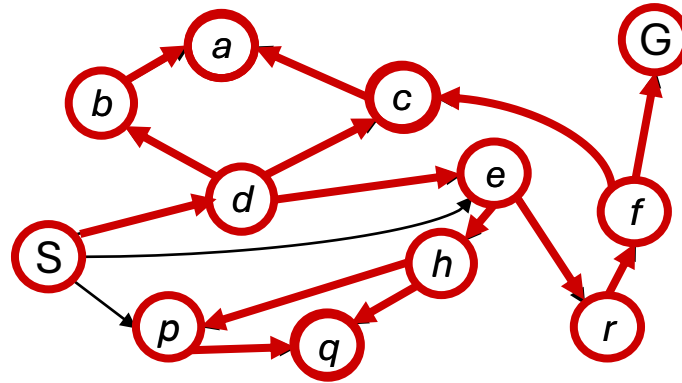
Depth-First Search

*Strategy: expand a
deepest node first*

Implementation:

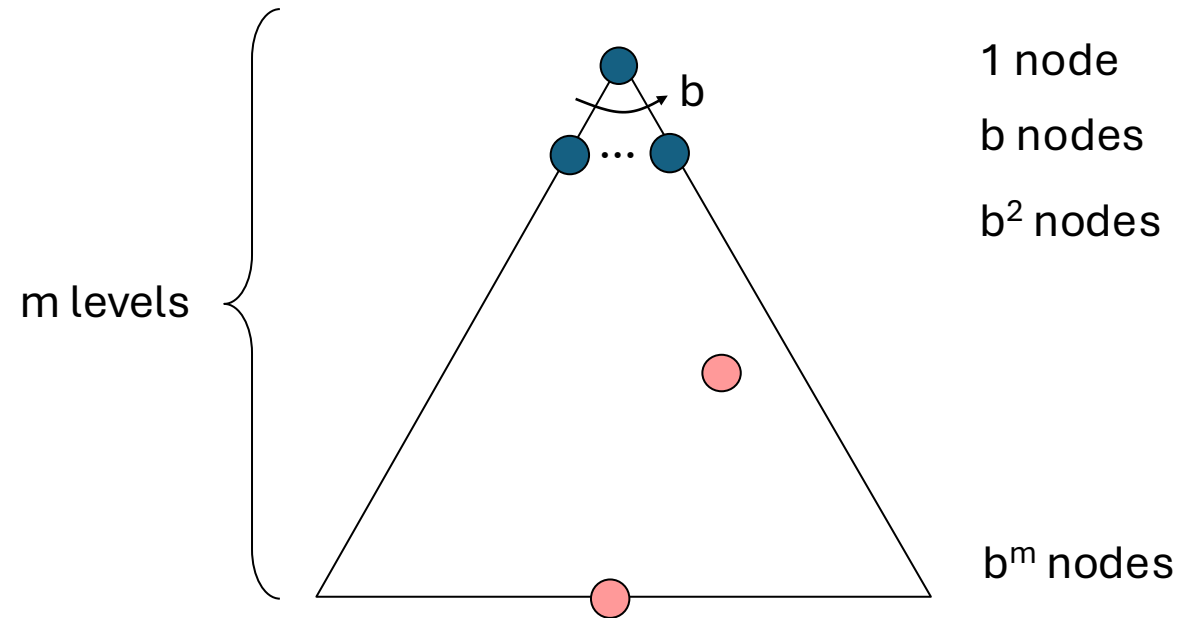
Fringe is a LIFO stack

(successors go on top)



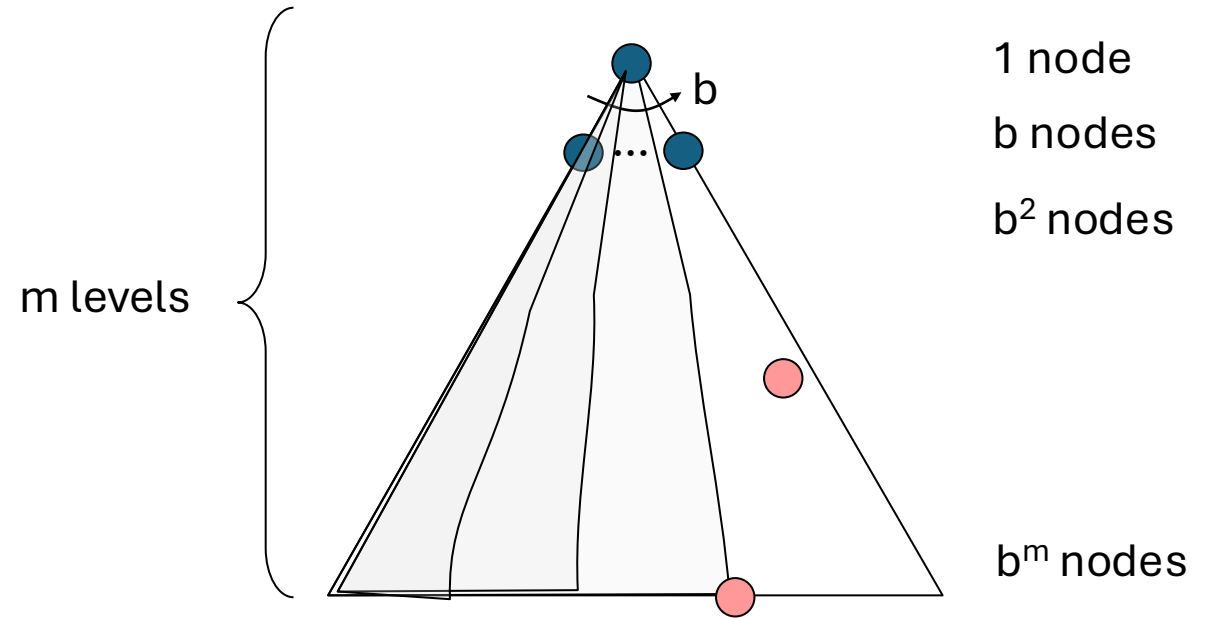
Search Algorithm Properties

- Complete: Guaranteed to find a solution if one exists?
- Optimal: Guaranteed to find the least cost path?
- Time complexity?
- Space complexity?
- Cartoon of search tree:
 - b is the branching factor
 - m is the maximum depth
 - solutions at various depths
- Number of nodes in entire tree?
 - $1 + b + b^2 + \dots + b^m = O(b^m)$



Depth-First Search (DFS) Properties

- What nodes DFS expand?
 - Some left prefix of the tree.
 - Could process the whole tree!
 - If m is finite, takes time $O(b^m)$
- How much space does the fringe take?
 - Only has siblings on path to root, so $O(bm)$
- Is it complete?
 - m could be infinite, so only if we prevent cycles (more later)
- Is it optimal?
 - No, it finds the “leftmost” solution, regardless of depth or cost

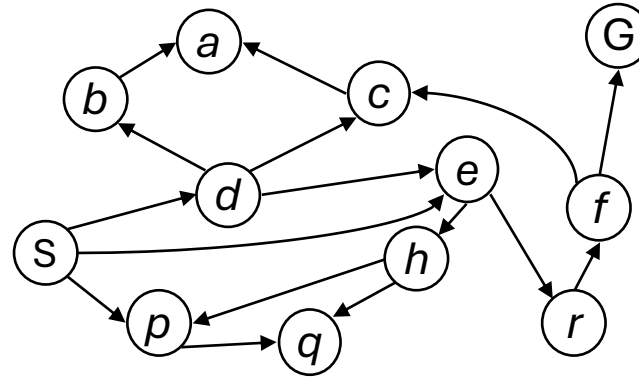


Breadth-First Search

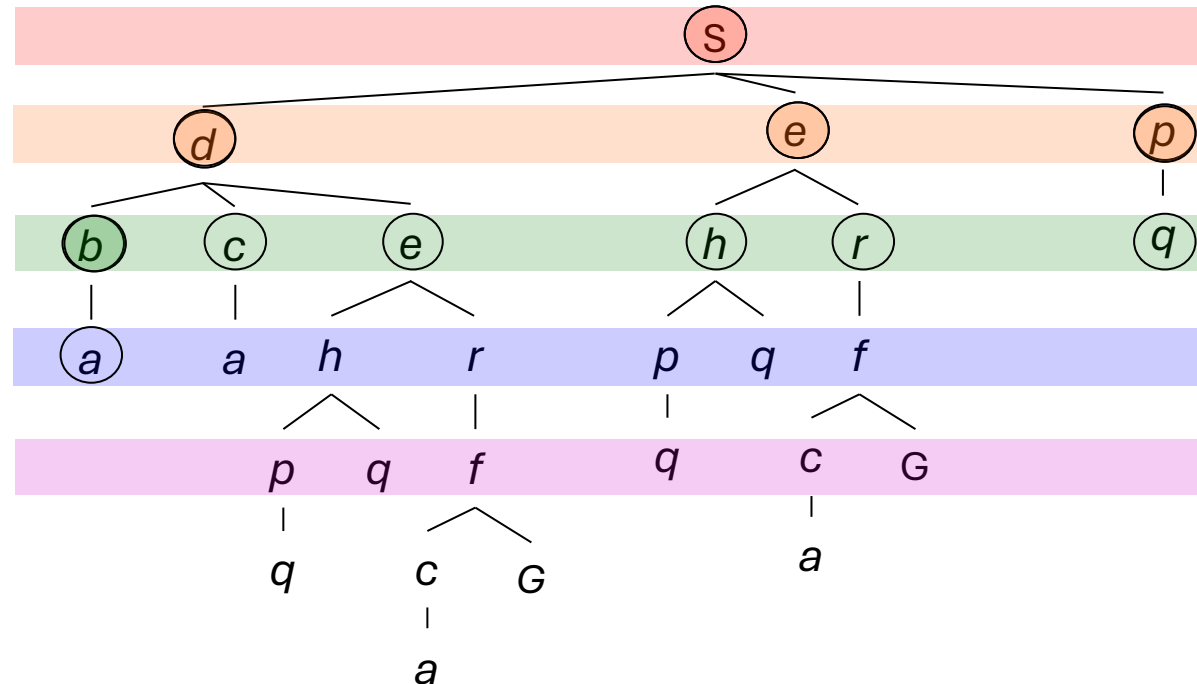
Strategy: expand a shallowest node first

Implementation: Fringe is a FIFO queue

(successors go at end)



Search
Tiers

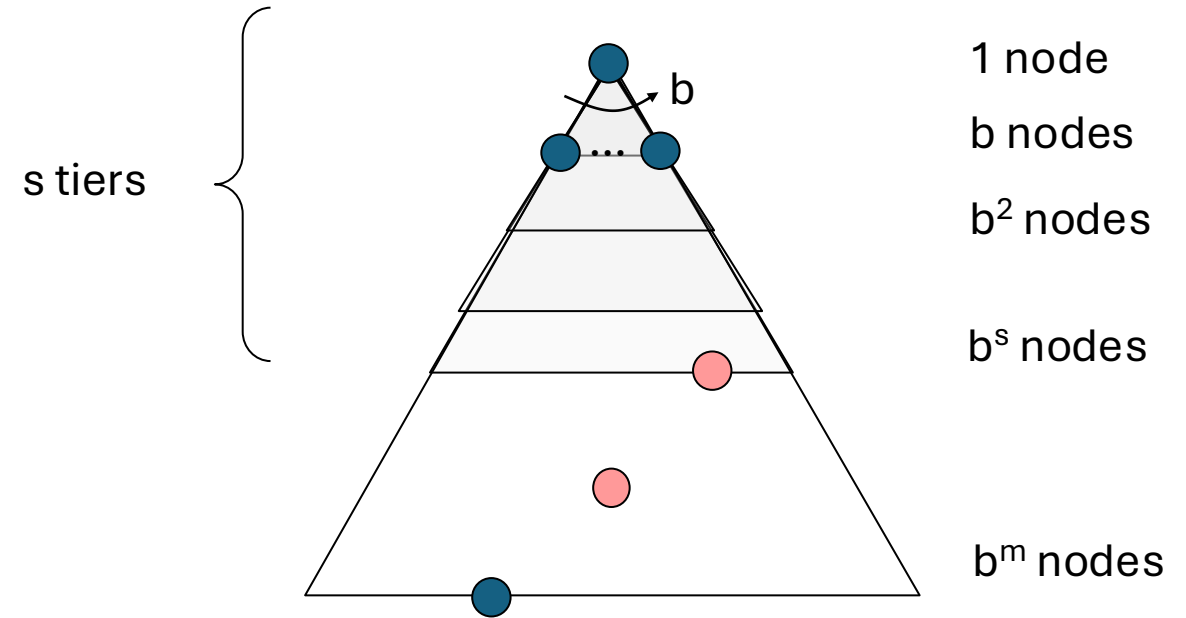


Analysis of BFS

- (See the textbook)
- Time complexity: $O(b^s)$, where s is depth of shallowest goal node
- Space complexity: $O(b^s)$
- Complete?
- Optimal?

Breadth-First Search (BFS) Properties

- What nodes does BFS expand?
 - Processes all nodes above shallowest solution
 - Let depth of shallowest solution be s
 - Search takes time $O(b^s)$
- How much space does the fringe take?
 - Has roughly the last tier, so $O(b^s)$
- Is it complete?
 - s must be finite if a solution exists, so yes!
- Is it optimal?
 - Only if costs are all 1 (more on costs later)



- Which would be better for a computational-space constrained scenario?
- Suppose solution is at depth $s \ll m$; $b > 2$. What would you use?

Iterative Deepening

- Idea: get DFS's space advantage with BFS's time / shallow-solution advantages
 - Run DFS with depth limit 1. If no solution...
 - Run DFS with depth limit 2. If no solution...
 - Run DFS with depth limit 3.

- Why is this a good idea?

$$b = 10, m = 5$$

$$N(IDS) = 5 \cdot 10 + 4 \cdot 100 + 3 \cdot 10^3 + 2 \cdot 10^4 + 10^5$$

- BFS generates successor nodes at the next level (can be fixed: how?)
 $(s+1) \cdot 1 + s \cdot b + (s-1) \cdot b^2 + \dots + 1 \cdot b^s \sim O(b^s)$
 $N(BFS) = 10 + 100 + 10^3 + 10^4 + 10^5 + 999,990$

- Space usage $O(bs)$ vs $O(b^s)$

Cost-Sensitive Search

- BFS expands the graph “uniformly” in terms of depth
- All nodes at a lower depth are expanded before nodes at a greater depth
- Often, actions (edges) have heterogeneous costs, so expanding lower depths may yield expensive solutions
- Can BFS be extended to expand lower-cost nodes first?

Yes!

