Lecture 23

CS 6380 Artificial Intelligence

First-Order Logic: Universe and Interpretation

Department of Computer Science and Engineering

Indian Institute of Technology Madras (IITM), India



Some material is based on lecture slides for CSE 571: Artificial Intelligence by Siddharth Srivastava, ASU.

Some of the deepest problems of our time…
Do goblins wear suits?













Do Goblins wear suits?
Are elves tiny?
Does the Dark Lord like sticks?

Universe



DarkLord



Dobby



Griphook



Let’s check other sources…

Do Goblins wear ~~suits~~?
Are elves ~~tiny~~ the skateboarding elite?
Does the Dark Lord like ~~sticks~~ jewelry?

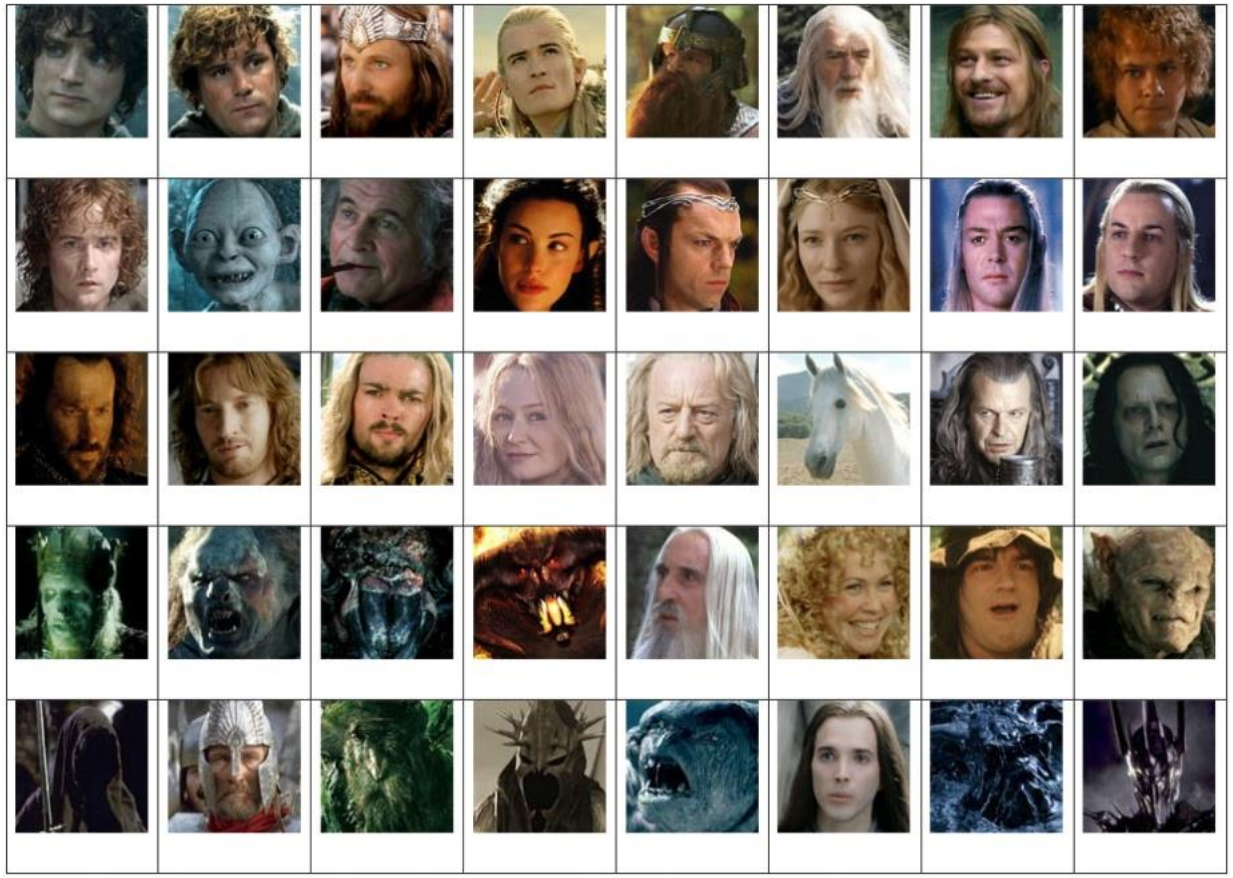
DarkLord



Legolas



Universe



The Great Goblin



Problems with Propositional Logic

- For some propositions, e.g. “Elves are tiny” each fictional world gives a clear truth assignment
- For others, e.g., “Dark Lords like sticks”, there’s no clear answer even within a single fictional world
- We can handle the first problem within propositional logic
- For the second, we need a richer representation: “Dark Lords like all sticks” vs “Dark Lords like some sticks”
- We also want to be able to reason about generalization and specialization

  Legolas likes skateboarding; Legolas is an elf. Thus: Some elves like skateboarding

  All elves like socks. Dobby is an elf. Thus, Dobby likes socks.

Different Worlds and Their Interpretations

Vocabulary

Names of things we want to talk about

- Constants: {DarkLord, HarryPotter, Dobby, Griphook}
- Predicates: {IsGoblin, IsElf, LikesGold}

Relationships we want to talk about

Universe



Interpretations of the Vocabulary

Dobby



IsElf() = True

DarkLord



Griphook



IsGoblin() = True

Statements

$[Goblins\ like\ gold] = ?$

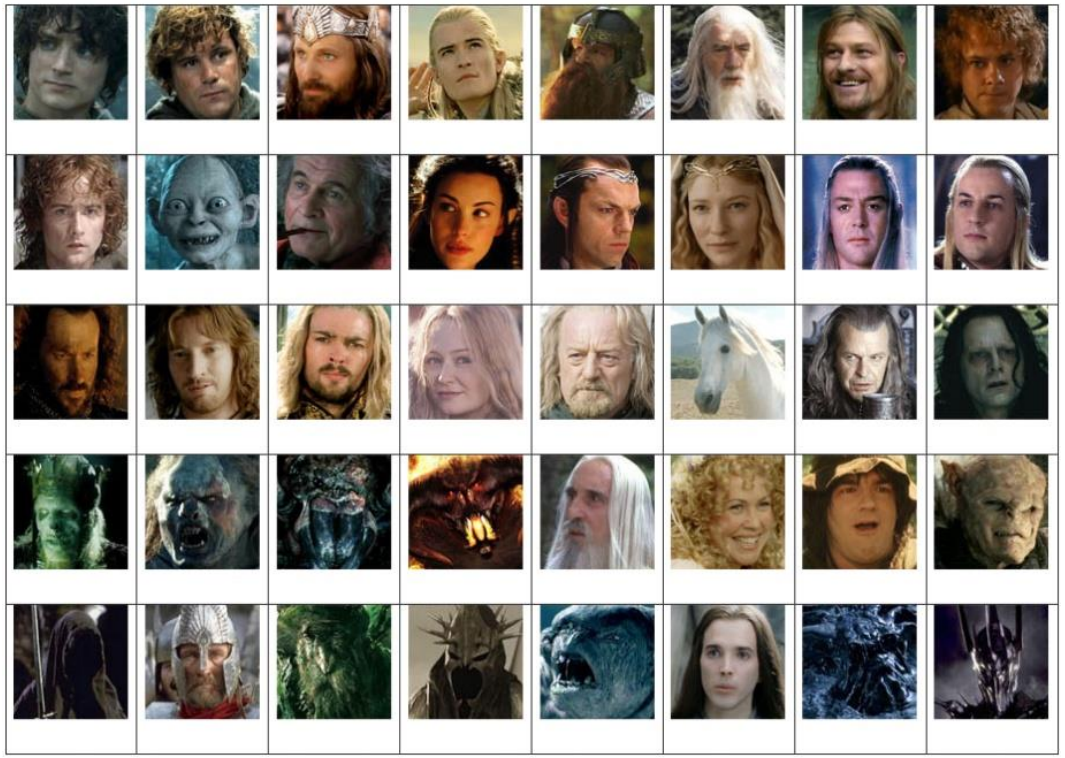
$[DarkLord\ likes\ sticks] = ?$

Different Worlds and Their Interpretations

Vocabulary

- Constants: {DarkLord, TheGreatGoblin, Legolas,…}
- Predicates: {IsGoblin, IsElf, LikesGold}

Universe



Interpretations of the Vocabulary

Legolas



IsElf() = True

DarkLord



The Great Goblin



IsGoblin() = True

Statements

$[Goblins\ like\ gold] = ?$

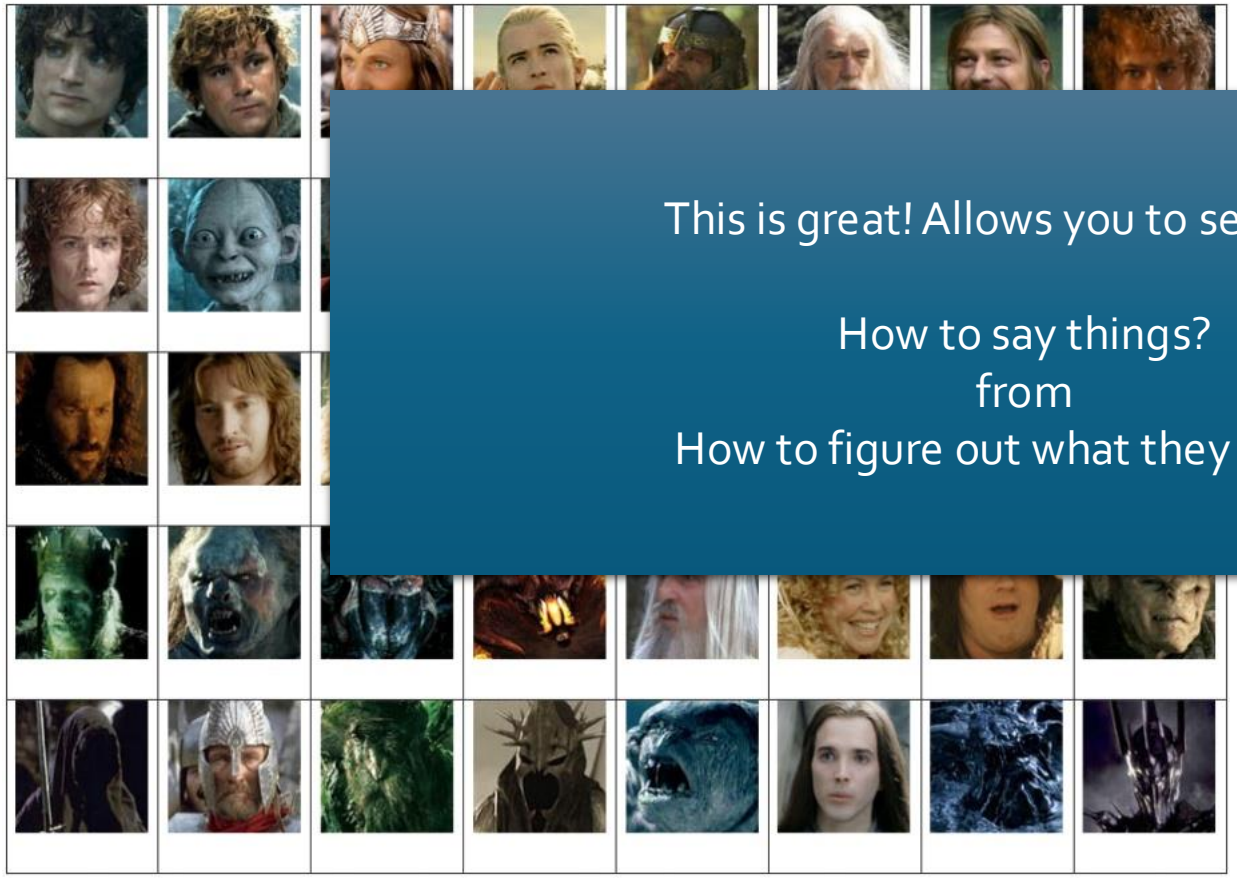
$[DarkLord\ likes\ jewelry] = ?$

Different Worlds and Their Interpretations

Vocabulary

- Constants: {DarkLord, TheGreatGoblin, Legolas,…}
- Predicates: {IsGoblin, IsElf, LikesGold}

Universe



Interpretations

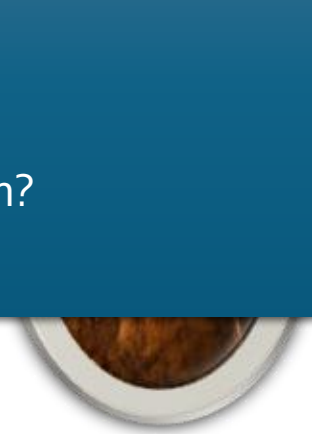
Legolas



IsElf() = True

DarkLord





IsGoblin() = True

This is great! Allows you to separate:

How to say things?
from
How to figure out what they mean?

Statements

$[Goblins\ like\ gold] = ?$

$[DarkLord\ likes\ jewelry] = ?$

First-Order Logic: Syntax

- Constants $Chennai, Madras, AI, IITM, Berkeley, Alfred$, 1, 2.., 100, ...
- Predicates $Studies(x, y), HasHotSummers(x), GoodAt(x, y), Attends(x, y)$
- Functions $Location(x), +$

Vocabulary

- Variables $x, y, a, b, ...$
- Connectives $\neg, \wedge, \vee, \rightarrow$
- Equality $=$
- Quantifiers $\forall, \exists$

Atomic Sentences in FOL

- Simple sentences: atomic sentences (a relation with constants or function application terms for arguments)
  - Studies(Alfred, AI)
  - Studies(DaughterOf(Alfred), AI)
  - HasHotSummers(Chennai)

- General form of atomic sentences
  - $Predicate(term_1, term_2, \ldots)$
  - $term_1 = term_2$
- Terms:
  - $constant, or$
  - $variable, or$
  - $function(term_1, term_2, \ldots)$

Complex Sentences

- We can combine atomic sentences as in propositional logic

$$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$$

- $Studies(Alfred, AI) \rightarrow GoodAt(Alfred, AI)$
- $(Location(IITM) = Chennai) \vee (Location(IITM) = Madras))$

...

Semantics: Welcome to the Mathematical Multi-Verse

- As in propositional logic, truth of FOL sentences is evaluated w.r.t. a model
- A model for FOL has much more information
  - A *universe* U of elements
  - An *interpretation* I that maps
    - Each *constant symbol* in the vocabulary to an element in U
    - Each *predicate symbol* in the vocabulary to a relation over U (of the same arity)
    - Each *function symbol* in the vocabulary to a function over U
- An interpretation *interprets* functions, predicates and constants in the vocabulary with properties of the universe

$V: R = \{Studies(x, y)\}; F = \{Location(x)\}; C = \{Alfred, AI, Madras, IITM\}$

$Model$:

$Universe = \{FetchRobot, AIBook, BiologyBook, CityOfChennai, MadrasOregon, Univ1\}$

$Interpretation$:

$[Location( )]_M = ?$ $[Studies( )]_M = ?$

$[Alfred]_M = ?; [AI]_M = ?; [Madras]_M = ?; [IITM]_M = ?$

FetchRobot



$[Studies(Alfred, AI)]_M = ?$

MadrasOregon



CityOfChennai

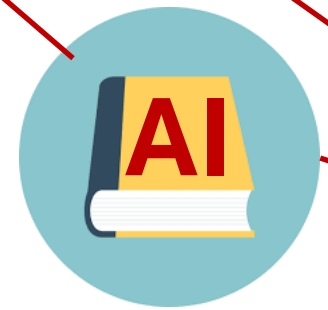
$Holding$

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AIBook



BiologyBook



Univ1

The Modeling Game

$V: R = \{Studies(x, y)\}; F = \{Location(x)\}; C = \{Alfred, AI, Madras, IITM\}$

$Model:$

$U = \{FetchRobot, AIBook, BiologyBook, CityOfChennai, MadrasOregon, Univ1\}$

$\ell:$

$[Location()]_M = \ell(); \quad [Studies()]_M = Holding()$

$[Alfred]_M = FetchRobot; [AI]_M = Book; [Madras]_M = CityOfChennai; [IITM]_M = Univ1$

$\ell(FetchRobot) = CityOfChennai; \ell(Univ1) = CityOfChennai, \ldots;$

$Holding(FetchRobot, AIBook) = True;$



Holding

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Alfred

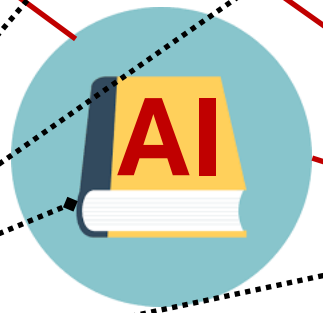
Studies

Location

AI

Madras

IITM



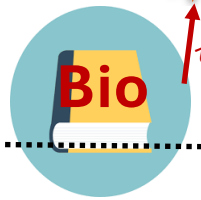
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MadrasOregon

CityOfChennai

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Univ1

The Modeling Game

$V: R = \{Studies(x, y)\}; F = \{Location(x)\}; C = \{Alfred, AI, Madras, IITM\}$

Sentences must use terms from the vocabulary, because models and interpretations can vary

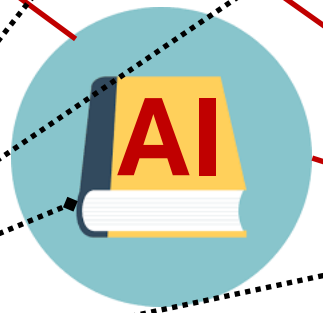
$[Location(Alfred) = Madras]_M = True\ or\ False?$
$[Location]([Alfred]) = [Madras]?$
$\ell(FetchRobot) = CityOfChennai?$



Holding

Alfred
Studies
Location
AI
Madras
IITM

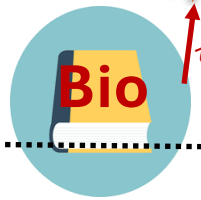
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MadrasOregon

CityOfChennai





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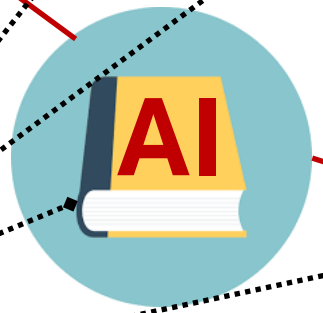
$$[Location(Alfred) = Madras]_M = True\ or\ False?$$



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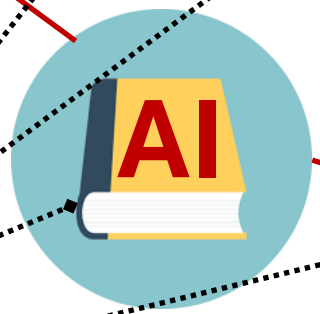
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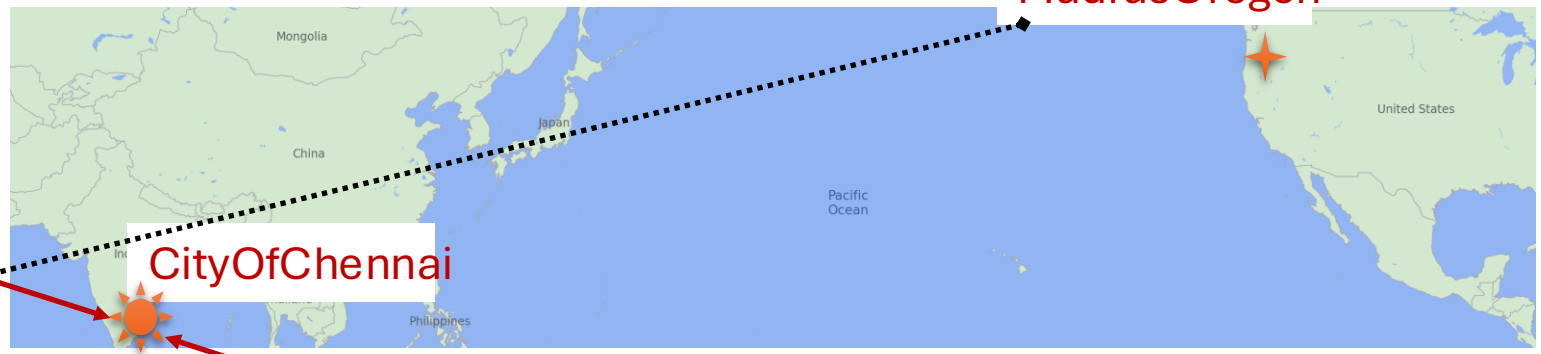


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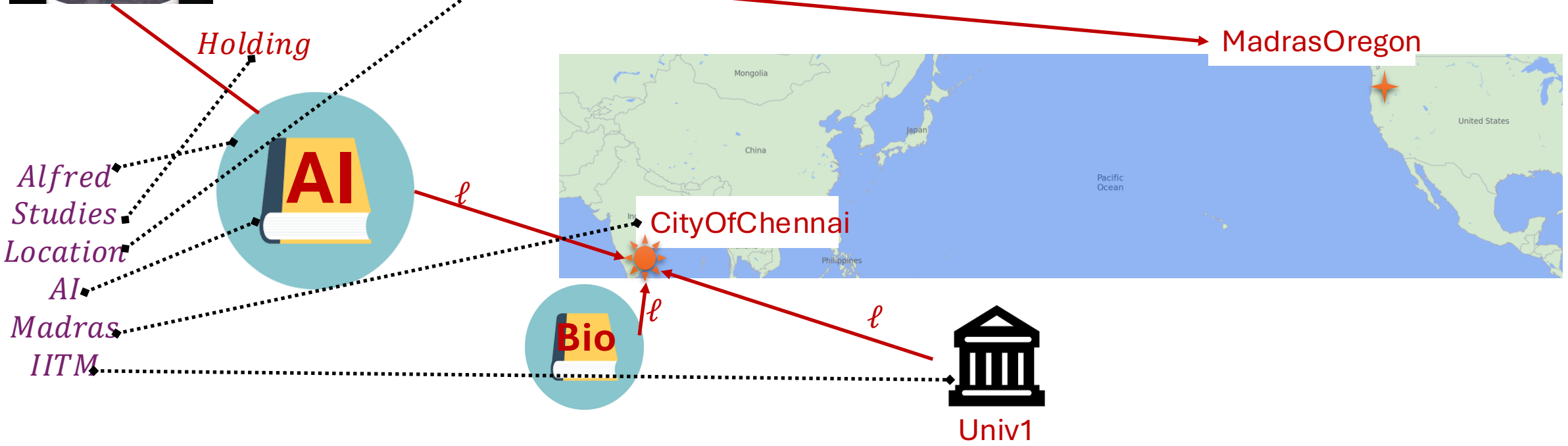
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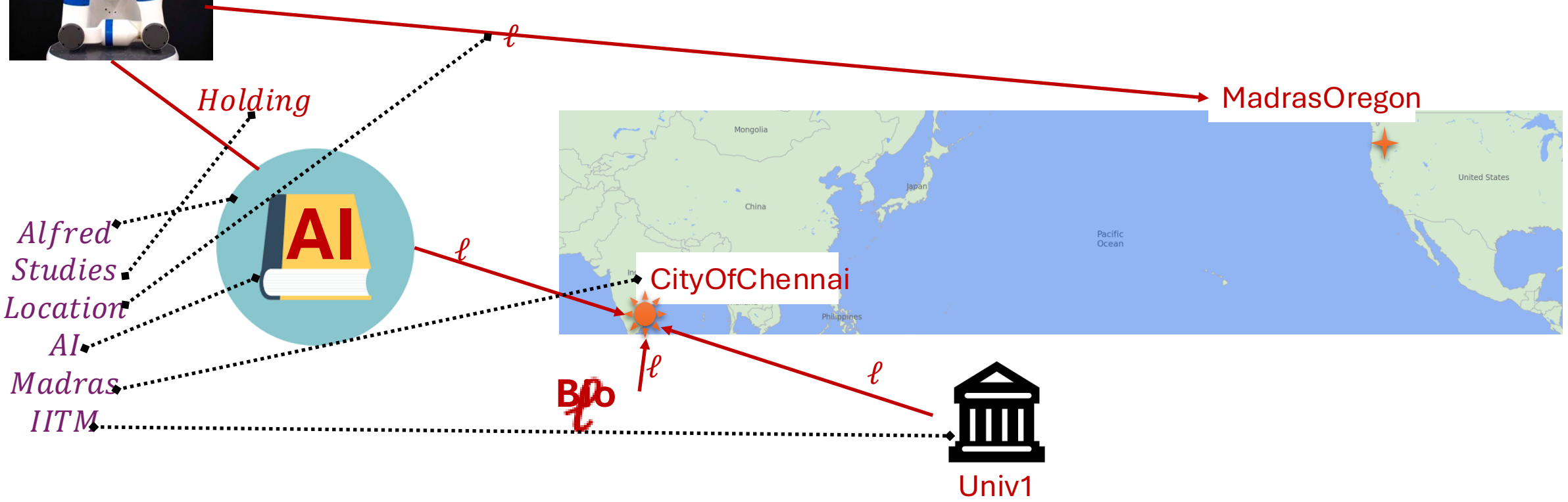
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Sentences must use terms from the vocabulary, because models and interpretations can vary

$$[Studies(Alfred, AI)]_M = True\ or\ False?$$





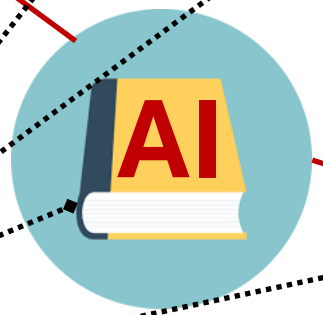
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$[Studies(Alfred, AI)]_M = True\ or\ False?$









Holding

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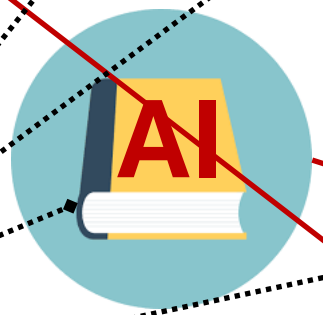
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Holding

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The Modeling Game

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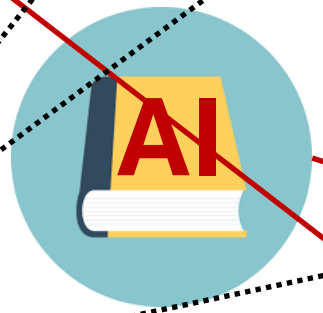


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Holding

MadrasOregon

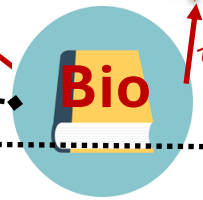
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CityOfChennai



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First-Order Logic: Fundamental Concepts

- Key takeaways:
  - Vocabulary is used to write sentences
  - Models (universe + interpretation) define meaning
  - Relations, constants, functions make the language much more expressive

- "There is a student at IIT Madras who is good in AI"

$$\exists x \left(Attends(x, IITM) \wedge GoodAt(x, AI) \right)$$

$\exists x\ \phi(x)$ is true in a model $M = < U, I >$ iff $\phi(c)$ is true in M for some $c \in U$

First-Order Logic: Fundamental Concepts

- Key takeaways:
  - Vocabulary is used to write sentences
  - Models (universe + interpretation) define meaning
  - Relations, constants, functions make the language much more expressive

In addition, FOL has supports specialization and generalization through quantifiers

Quantifiers In First-Order Logic

- Key takeaways:
  - Vocabulary is used to write sentences
  - Models (universe + interpretation) define meaning
  - Relations, constants, functions make the language much more expressive

- "There is a student at IIT Madras who is good in AI"

$$\exists x \left(Attends(x, IITM) \wedge GoodAt(x, AI) \right)$$

Semantics:

$\exists x\ \phi(x)$ is true in a model $M = < U, I >$ iff $\phi(c)$ is true in M for some $c \in U$

Quantifiers In First-Order Logic

- Key takeaways:
  - Vocabulary is used to write sentences
  - Models (universe + interpretation) define meaning
  - Relations, constants, functions make the language much more expressive

- But what about "all students at IIT Madras are good in AI"?

$$\forall x \left(Attends(x, IITM) \rightarrow GoodAt(x, AI) \right)$$

Semantics:

$\forall x \ \phi(x)$ is true in a model $M = < U, I >$ iff $\phi(c)$ is true in M for all $c \in U$

Quantifiers in First-Order Logic

- Constants: {DarkLord, HarryPotter, Dobby, Griphook}
- Predicates: {IsGoblin, IsElf, IsStick, Likes}

- Express in FOL:

The Dark Lord likes all sticks

The Dark Lord likes some sticks

The Dark Lord likes a unique stick

Interpretations

DarkLord



Dobby



IsElf() = True

Griphook



IsGoblin() = True

Statements

$[Goblins\ like\ gold] = ?$

$[DarkLord\ likes\ sticks] = ?$

Fun With Quantifiers

- All IIT Madras students study AI
- All AI students study at least one other subject
- There is a subject that no one studies