Lecture 20

CS 6380 Artificial Intelligence

Propositional Logic

Department of Computer Science and Engineering

Indian Institute of Technology Madras (IITM), India



Some material is based on lecture slides for CSE 571: Artificial Intelligence by Siddharth Srivastava, ASU.

Course Logistics

Logistics

- HW3-Quiz moved to September 22, Tuesday

- User Study on Robotics Representations
  - Does not have any effect on your grades.
  - Completely independent of the course
  - Participation is voluntary
  - Rs. 100 compensation for about 30-45 min of your time

Back to AI

A Comparison of the Three Representational Choices

$\text{At_}t_1 Phoenix_At_t_2 Ontario_\ldots$

Atomic Representation

- A state is a black-box
- String representations are used to alleviate the situation

$loc_{t_1} = Phoenix$
$loc_{t_2} = Ontario$

Factored Representation

- A state is a valuation of a fixed set of "state variables"
- Successor functions for each action can depend on a small set of variables
- Problem: dependent on object names, numbers

$loc(t_1) = Phoenix$
$loc(t_2) = Ontario$

Relational Representation

- A state is defined using properties and relationships among objects
- Specifying successor functions *seems to be easier* through the use of variables, quantifiers

Underlying problem: building a formal system of reasoning with symbolic representations

Propositional Logic: Syntax

- Syntax: How to state something
- Semantics: What a statement means

The proposition symbols P_1, P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence (negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Propositional Logic

- Syntax: how to make well-structured statements
- Semantics: determines which statements are true

P_1 = "It rains a lot in Tempe"
P_2 ="The university in Tempe has more than 100 AI students"

- P_1 and P_2 are propositions
- Are they true?
- How can we determine truth?

Dictionary Definitions

"Fact"

OED:
a thing that is known to be true, especially when it can be proved

M-W:
a piece of information presented as having objective reality

"Proposition"

Something offered for consideration or acceptance

Propositional Logic: Semantics

P_1 = "It rains a lot in Tempe"

P_2 = "The university in Tempe has more than 100 AI students"

Models are like possible worlds/interpretations

- Model M_1
  - $[P_1]_{M_1} = False$
  - $[P_2]_{M_1} = True$
- Another model: M_2
  - $[P_1]_{M_2} = True$
  - $[P_2]_{M_2} = False$



The vale of Tempe (note: no university)

- A model assigns a truth value to every proposition

Validity in Propositional Logic

- In mathematical logic, validity supersedes truth
- A sentence is valid if it is true in all possible models
  - You are familiar with truth tables: every row in a truth table is a model

- Scientific notions of truth try to achieve this by conducting experiments
  - Like testing under different models, but sometimes not possible to enumerate all possibilities

Model Theoretic "Truth"

- Empirical truth relies on sampling, or conducting multiple observations to determine truth of propositions
- But science goes beyond tabulating observations
  - Main role of science: predictions!
    - In AI: what would happen if self-driving car changed lanes now?
  - Empirical determination of truth is not sufficient for making predictions
  - Need to build a theory: inductive reasoning
  - Then use the theory to make predictions: deductive reasoning
  - Science = predictive, empirically verifiable theory

Theories in Mathematical Logic

- Mathematical logic also allows the expression of theories, and automation of reasoning – the core of what an AI agent needs to do

- Theory = collection of logical sentences (a conjunction of formulas)
- E.g., P_1: "AI is a hot topic"; P_2: "A university in town is good in AI"

"P_3: town has hot summers"

Theory:

$$T = \{P_1 \wedge P_3 \rightarrow P_2\}$$

Theories in Mathematical Logic

- E.g., P_1: "AI is a hot topic"; P_2: "A university in town is good in AI"

"P_3: town has hot summers"

Theory:

$$T = \{P_1 \wedge P_3 \rightarrow P_2\}$$

- A theory T is said to be consistent with a model M iff $M \models T$
- A theory is said to be logically consistent iff it doesn't imply a contradiction
  - A contradiction is any statement of the form $P \wedge \neg P$

Theories in Mathematical Logic

$$T = \{P_1 \wedge P_3 \rightarrow P_2\}$$

- A theory is said to be consistent with a model M iff $M \vDash T$
- A theory is said to be logically consistent iff it doesn't imply a contradiction
  - A contradiction is any statement of the form $P \wedge \neg P$

$$T' = \{P_1 \wedge P_3 \rightarrow P_2, \qquad P_1 \rightarrow \neg\, P_2\}$$

Is T' logically consistent?

Theories in Mathematical Logic

- E.g., P_1: "AI is a hot topic"; P_2: "A university in town is good in AI"

"P_3: town has hot summers"

Theory:

$$T = \{P_1 \wedge P_2 \rightarrow P_3\}$$

Is T consistent with these models?

- $M_{Tempe,AZ} : P_1 \leftarrow True; P_2 \leftarrow True; P_3 \leftarrow True$
- $M_{Tempe,Greece}: P_1 \leftarrow True; P_2 \leftarrow False; P_3 \leftarrow False$
- $M_{Toronto}$:P_1: $\leftarrow True; P_2 \leftarrow True; P_3 \leftarrow False$

Notation:
If $[\varphi]_M = true$, we say M "models" φ
"$M \vDash \varphi$"

A theory T is said to be consistent with a model M iff $M \vDash T$

In Class Exercise. A Theory About IIT Madras

- Let's build a small theory using a place everyone in this room knows well, IIT Madras in Chennai
- Propositions
  - P_1 is "Chennai has a hot and humid climate"
  - P_2 is "IIT Madras is strong in AI research"
  - P_3 is "A university in a hot city can still be strong in AI"
- Theory $T = \{P_1, P_3 \rightarrow P_2\}$
- Task. In pairs, write two more propositions about IIT Madras and one implication that connects them

Checking Consistency Across Models

- We test our theory the same way we tested the Tempe theory earlier, by proposing models and checking whether the implication holds
  - Model M_1, the actual IIT Madras campus. P_1 true, P_2 true, P_3 true
    - This model satisfies T, since the antecedent and the consequent are both true
  - Model M_2, a university in a cool hill town. P_1 false, P_2 true, P_3 true
    - This model also satisfies T, since the antecedent is already false
  - Model M_3, a rival campus. P_1 true, P_2 false, P_3 true
    - Work out by hand whether this model is consistent with T, and explain your answer in one sentence
- Task for the class. Which of these three models are consistent with T, and why

Where Propositional Logic Runs Out

- P_2 treated "IIT Madras is strong in AI research" as one indivisible block
- Suppose we now want to talk about many objects at once, such as "every department at IIT Madras publishes in top venues"
- Or about at least one object, such as "some hostel at IIT Madras has the best food on campus"
- Propositional logic has no way to open up an object and talk about its properties one at a time
- To do this we need variables, predicates and quantifiers. This is exactly where First Order Logic begins

In Class Exercise Two. From Propositions to Predicates

- Let's rewrite pieces of our IIT Madras theory using objects and predicates instead of whole propositions
  - Constants. Chennai, IITMadras, HostelGanga, HostelCauvery
  - Predicates. IsHumid(x), IsStrongInAI(x), HasGoodFood(x), IsHostel(x)
- Try writing "Every hostel at IIT Madras has decent food" using a quantifier and a predicate
- Try writing "Some hostel at IIT Madras has the best food on campus" the same way
- Keep your answers. We will formalize exactly this kind of sentence when we cover First Order Logic syntax

We now know how to create representations...
How do we use them?

Can We Automate Reasoning?

- If it's raining, it's not sunny. Your solar-powered car needs sunlight to charge.

- It's raining. Is your car charging?

Can We Automate Deductive Reasoning?

- If it's raining, it's not sunny. Your solar-powered car needs sunlight to charge.
- It's raining. Is your car charging?
- Did you use a truth table to answer this?

Can We Automate Deductive Reasoning?

- Can we build an automated inference procedures to derive *all true implications of a theory??*
- Huge applications in science, mathematics and AI
  - E.g., if we can write down a logical theory of mathematics, all theorems can be computed automatically!
  - Newton's laws of motion + optics, in a self-driving car: compute all safe paths!

Can We Automate Deductive Reasoning?

- Can we build an automated inference procedures to derive *all true im*
- Hu
  - n be
  - aths!

That's the lure of mathematical logic!

In Class Exercise. More Puzzles Like This One

- Same drill as the solar-powered car. Read each pair of sentences, work out the answer, then notice what pattern you used
- If there is a fest at IIT Madras, the hostels get a late curfew. Tonight there is a fest. Do the hostels get a late curfew
- If the shuttle is late, students miss the first class. The shuttle is on time today. Do students miss the first class
- If it is exam season, the library stays open all night. It is exam season. Is the library open all night
- Try answering all three in under a minute, without drawing a truth table
- Which of these three could you not answer with full confidence, and why?

If there is a fest at IIT Madras, the hostels get a late curfew. Tonight there is a fest. Do the hostels get a late curfew

- Let P be "there is a fest," Q be "the hostels get a late curfew."
- Given: $P \rightarrow Q$, and P.
- By Modus Ponens ($P \rightarrow Q$, P, therefore Q): **Q is true.**
- Hence, the hostels get a late curfew.

If the shuttle is late, students miss the first class. The shuttle is on time today. Do students miss the first class

- Let P be "the shuttle is late," Q be "students miss the first class."
- Given: $P \rightarrow Q$, and $\neg P$ (the shuttle is on time, so P is false).
- By Modus Ponens ($P \rightarrow Q$, P, therefore Q)
- Since $\neg P$, you cannot conclude about Q.

If it is exam season, the library stays open all night. It is exam season. Is the library open all night

- Let P be "it is exam season," Q be "the library stays open all night."
- Given: $P \rightarrow Q$, and P.
- By Modus Ponens ($P \rightarrow Q$, P, therefore Q): **Q is true.**
- Hence, the library is open all night.

The Pattern You Just Used

- The solar car question and the first two IIT Madras questions all had the same shape
- From “if P then Q” and “P”, conclude “Q”. This pattern is called Modus Ponens
- The shuttle question was different, since it told you the shuttle was on time, making the antecedent false. Modus Ponens needs the antecedent itself to be true, so that puzzle could not be answered the same way
- Every one of you found these answers instantly, without writing a truth table. Good news, that means the pattern is mechanical
- If a pattern is mechanical, a machine can apply it too. That is the whole idea behind a proof system
- A proof system is just a small set of mechanical patterns like Modus Ponens, applied one after another, that always take true statements to true statements