

CS 6380 Artificial Intelligence

Stochastic and Partially Observable Games

Department of Computer Science and Engineering
Indian Institute of Technology Madras (IITM), India



Games With Dice

Now We Break Another Assumption

- You roll before you move, so White knows White's own options and has no idea what Black will be allowed to do
- Why is a double half as likely as any other roll? Because there is only one ordered way to roll it

36

ordered ways to roll two dice, all equally likely

21

distinct rolls, since a 6 then 5 matches a 5 then 6

1 in 36

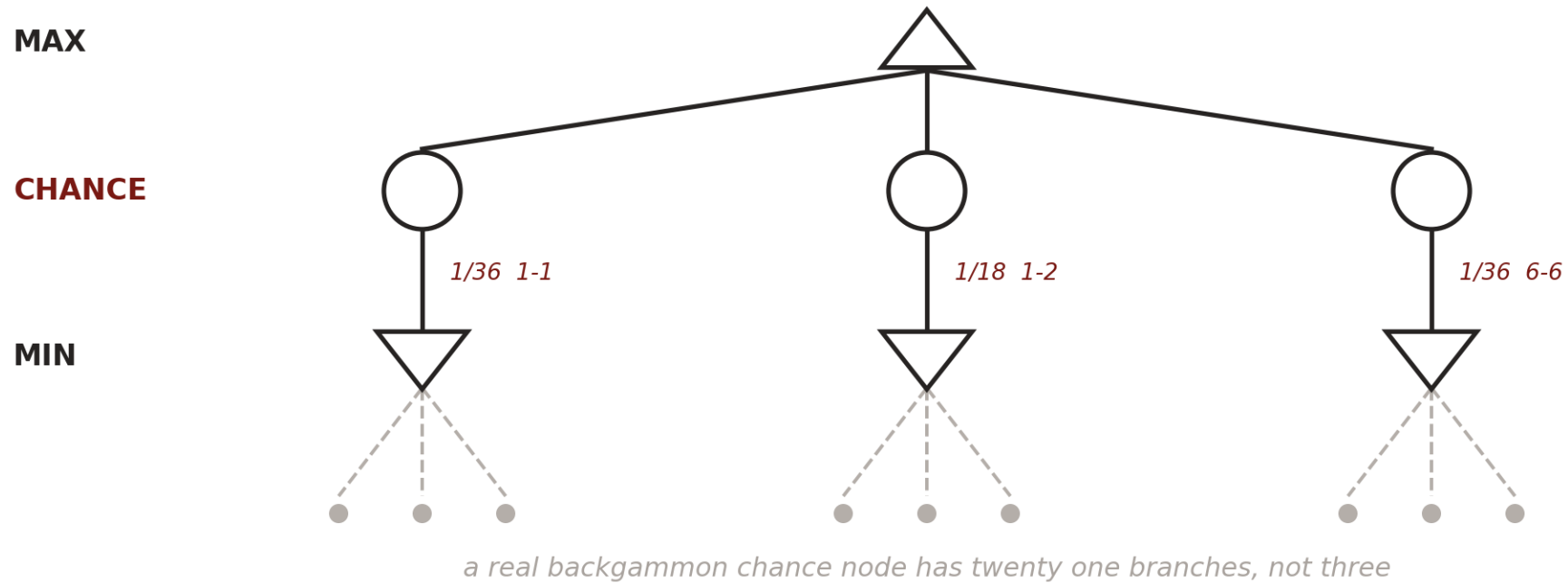
the probability of each of the six doubles

1 in 18

the probability of each of the other fifteen rolls

The Missing Layer, Chance Nodes

- Nobody chooses at a chance node, so it can take neither a max nor a min. It has to take an average



Expectiminimax, One Extra Case

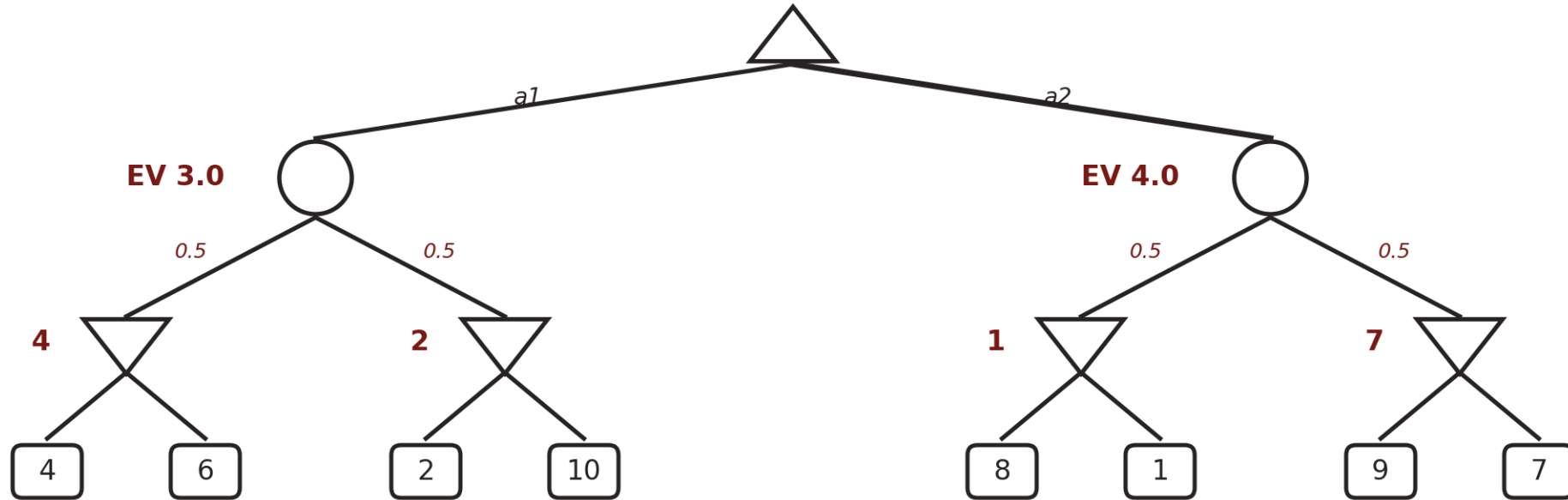
- Positions no longer have definite values, only expected values, so you can play perfectly and still lose

```
EXPECTIMINIMAX(s) =  
    UTILITY(s)                                if TERMINAL-TEST(s)  
    max over a of EXPECTIMINIMAX(RESULT(s,a))  if PLAYER(s) = MAX  
    min over a of EXPECTIMINIMAX(RESULT(s,a))  if PLAYER(s) = MIN  
    sum over r of P(r) x EXPECTIMINIMAX(RESULT(s,r))  if PLAYER(s) = CHANCE
```

RESULT of s and r is the same board as s, plus the knowledge that the roll came out r. Nothing moved, you only learned something.

Worked Example, Work It Bottom Up

- Two moves, a coin flip after each, then the opponent replies



MAX takes the larger expected value, so play a2

Your Turn, Then a Twist

- Move a1 rolls a die. Move a2 is a safe deterministic line worth exactly 1.5

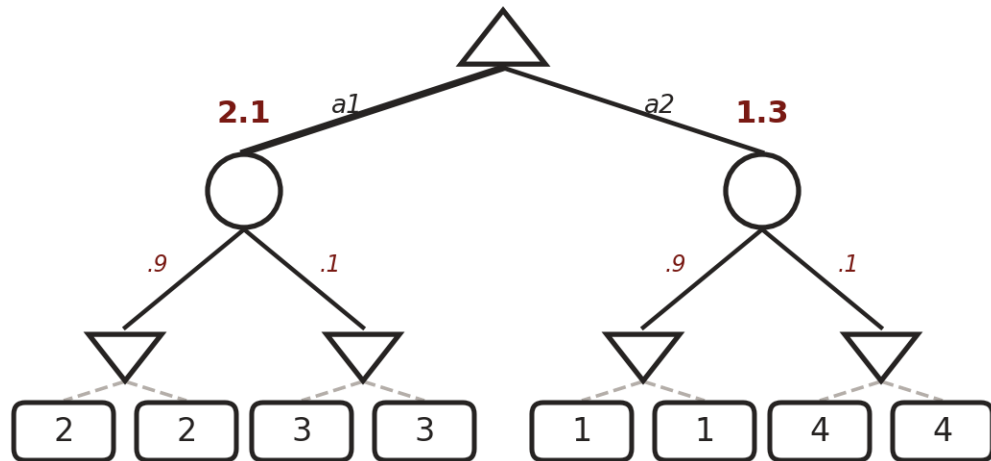
A quarter chance	of a good roll worth 8
A half chance	of a fair roll worth 2
A quarter chance	of a bad roll worth minus 4

$(0.25 \times 8) + (0.5 \times 2) + (0.25 \times -4) = 2$, which beats 1.5, so play a1
change the bad roll to -20 and it is -2, so now play a2 instead

One outcome with only a quarter probability moved, and the entire decision reversed. This is why a poker player folds a hand that usually wins.

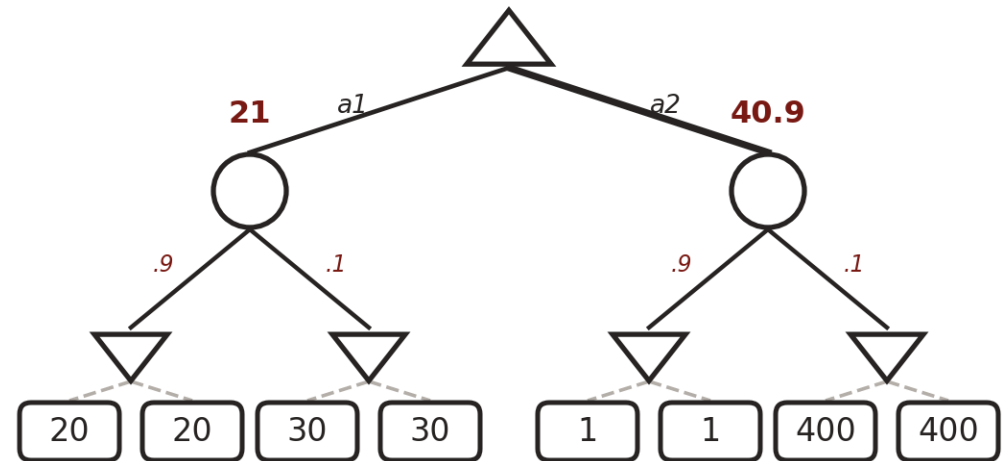
A Nasty Surprise About Scale

- The leaf ordering is identical in both trees. Only the spacing changed, and the best move flipped
- Max and min only compare, but an average does arithmetic, so it feels the gaps between values



leaf values 1, 2, 3, 4

a1 wins



same ordering, values 1, 20, 30, 400

a2 wins

The Price of Uncertainty

- The cost is b to the m times n to the m , where n is the number of distinct rolls. Three plies is all you get

Why alpha beta loses its grip

It works by ignoring futures that will not happen under best play. With dice, any sequence can happen if the dice cooperate, so there is very little to rule out.

What people do instead

Bound the utilities and you can bound an average without seeing every child. Or abandon analysis and run Monte Carlo rollouts, thousands of random games played out from here.

*Replacing analysis with a very large number of random games is about to take over the whole field.
Remember it.*

Games In the Dark

Now We Turn Off the Lights

- An illegal move announcement is not a wasted turn. It is free information about where the enemy is

Kriegspiel, chess in the dark

Each player sees a board holding only their own pieces. A referee sees everything and announces captures and checks along with their direction. Propose a move, and if it is illegal you are told so and may propose again, learning as you go.

Belief states

The set of every board consistent with everything you have perceived. A singleton at the start, twenty positions after Black replies once. Your evaluation function now includes its size, and smaller is better.

Knowing where you are is literally worth material.

Three Kinds of Checkmate

- Any predictable optimal strategy leaks information, so optimal play requires a willingness to be random

Guaranteed

Checkmate for every board in the belief state and every percept sequence. It must work even if the opponent can see all your pieces, which sounds harsh and simplifies everything.

Probabilistic

Works with probability 1 because you randomise. Chase a lone king with your own king at random and you must eventually bump into it.

Accidental

Works for some boards in the belief state and happens to land. Most checkmates between humans are of exactly this kind.

This is also why restaurant hygiene inspectors turn up unannounced.

Card Games and Averaging Over Clairvoyance

- Solve every possible deal as if it were fully observable, then play the move that does best on average

Day 1 Road A has gold. Road B forks. Left fork more gold, right fork a bus. So take B.

Day 2 Road A has gold. Road B forks. Right fork more gold, left fork a bus. So take B.

Day 3 Road A has gold. Road B forks. One fork has more gold, the other has the bus, and you do not know which.

B was right on Day 1 and right on Day 2, so B must be right on Day 3. Then you get hit by a bus.