

Lecture 5

CS 6260 Automated Planning and Learning

Classical Planning: Representation

Department of Computer Science and Engineering

Indian Institute of Technology Madras



Recap: A* Complexity

$O(b^{d \times \epsilon})$ $O(b^d)$ ~~$O(b^d)$~~

n h C^*

~~$h - C^n$~~ ~~$h - C^n$~~



A^* $C^* = 100$ $h(A) = 90$

$$f = g + \underline{h}$$

$$\frac{100 - 90}{100} \rightarrow 10$$

Classical Planning

State-Transition Systems in Planning

~~State-transition system~~ or classical planning domain:

$$\Sigma = (S, A, T, \text{cost}) \quad \text{or} \quad (S, A, T)$$

- S - finite set of states
- A - finite set of actions
- $T: S \times A \rightarrow S$

prediction (or state-transition) function

- *partial function*: $T(s, a)$ is not necessarily defined for every (s, a)
- a is *applicable* in s iff $T(s, a)$ is defined
- $\text{Domain}(a) = \{s \in S \mid a \text{ is applicable in } s\}$
- $\text{Range}(a) = \{T(s, a) \mid s \in \text{Domain}(a)\}$
- $C: S \times A \rightarrow \mathbb{R}^+$ or $C: A \rightarrow \mathbb{R}^+$
 - optional; default is $\text{cost}(a) \equiv 1$
 - money, time, something else

- *plan*: a sequence of actions $\pi = \langle a_1, \dots, a_n \rangle$
- π is *applicable* in s_0 if the actions are applicable in the order given

$$T(s_0, a_1) = s_1$$

$$T(s_1, a_2) = s_2$$

...

$$T(s_{n-1}, a_n) = s_n$$

- In this case define $T(s_0, \pi) = s_n$, and $\hat{T}(s_0, \pi) = \langle s_1, \dots, s_n \rangle$

- *Classical planning problem*:

- $P = (\Sigma, s_0, S_g)$
- *planning domain*, initial state, set of goal states

- *Solution for P* :

- a plan π such that that $T(s_0, \pi) \in S_g$

$$T(s, a) = s'$$
$$T(s, a) = s$$

$$s_n \in S_g$$

$$T(s, a, s') = p$$
$$0 < p < 1$$



Planning Problems

- *plan*: a sequence of actions $\pi = \langle a_1, \dots, a_n \rangle$
- π is *applicable* in s_0 if the actions are applicable in the order given

$$T(s_0, a_1) = s_1$$

$$T(s_1, a_2) = s_2$$

...

$$T(s_{n-1}, a_n) = s_n$$

- In this case define $T(s_0, \pi) = s_n$, and $\hat{T}(s_0, \pi) = \langle s_1, \dots, s_n \rangle$

- *Classical planning problem*:

- $P = (\Sigma, s_0, S_g)$
- planning domain, initial state, set of goal states

- *Solution for P* :

- a plan π such that that $T(s_0, \pi) \in S_g$

- *Solution for P* : a plan π such that that $T(s_0, \pi) \in S_g$
 - *Minimal solution*: no subsequence is also a solution
 - *Shortest solution*: no solution has fewer actions
 - *Optimal solution*: no solution has lower cost

- **Example**: Suppose P has three solutions

- $\pi_1 = \langle a_1 \rangle$
- $\pi_2 = \langle a_2, a_3, a_4, a_5 \rangle$
- $\pi_3 = \langle a_2, a_3, a_1 \rangle$

- Then π_1 is both shortest and optimal

$T(s_0, a_2) = s_{100}$
 $T(s_{100}, a_3) = s_{92}$
 $\vdots$
 $s \in S_g$

Which solutions are minimal?

A. π_1 B. π_2 C. π_3 D. A & B E. B & C

Acting with a Plan

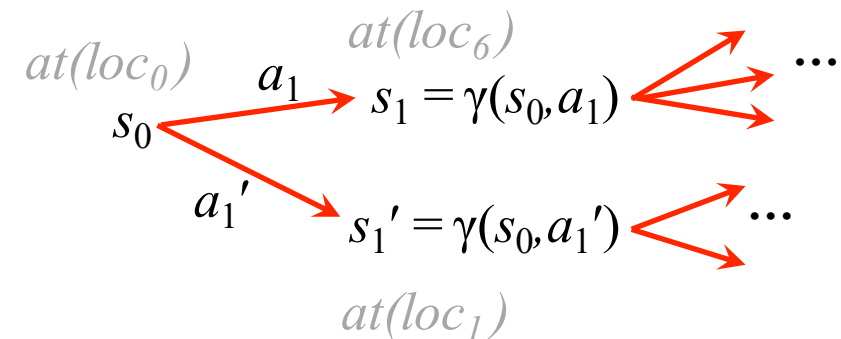
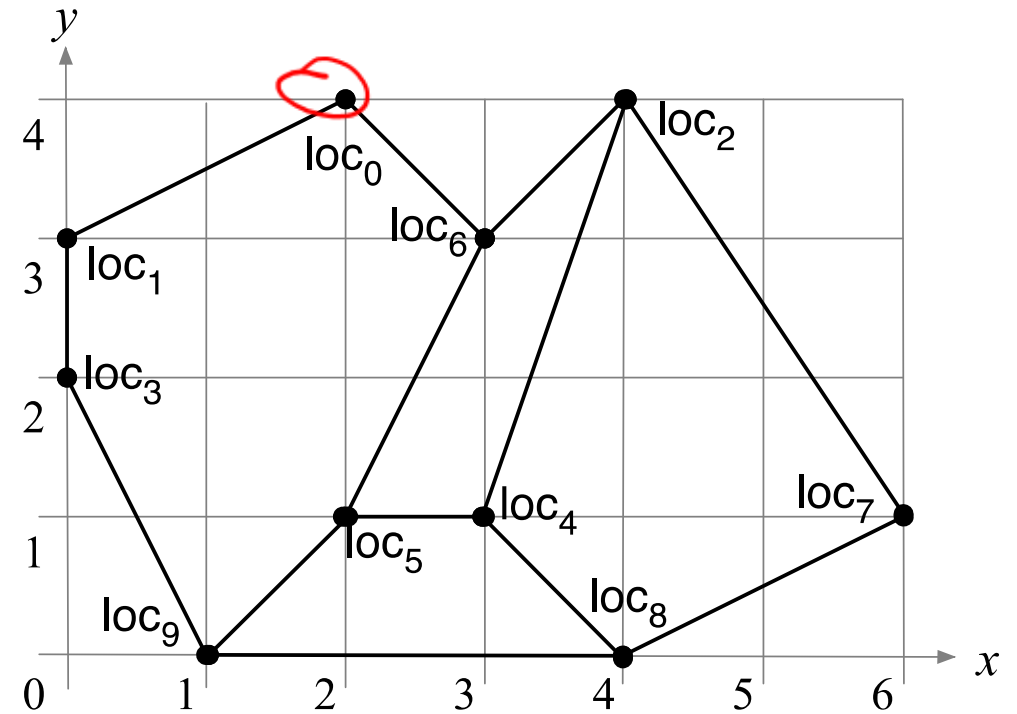
- Run-Plan(Σ, π):
 - **while** True **do**
 - 1 $s \leftarrow$ observe current state
 - if** $\pi = \langle \rangle$ **then**
 - 2 $\quad$ **return** success
 - $a \leftarrow \text{pop}(\pi)$
 - 3 **if** $a \notin \text{Applicable}(s)$ **then return** failure
 - perform action a
- Where π is a solution to the classical planning problem:
 - $P = (\Sigma, s_0, S_g)$
 - planning domain Σ , initial state s_0 , set of goal states S_g
 - Recall: $\Sigma = (S, A, T, C)$ or (S, A, T)

Acting with a Plan

- Run-Plan(Σ, π):
 - while** True **do**
 - $s \leftarrow$ observe current state
 - if** $\pi = \langle \rangle$ **then**
 - return** success
 - $a \leftarrow \text{pop}(\pi)$
 - if** $a \notin \text{Applicable}(s)$ **then return** failure
 - perform action a
- Where π is a solution to the classical planning problem:
 - $P = (\Sigma, s_0, S_g)$
 - planning domain Σ , initial state s_0 , set of goal states S_g
 - Recall: $\Sigma = (S, A, T, C)$ or (S, A, T)
- Ideally, Run-Plan($\Sigma, \langle a_1, \dots, a_n \rangle$) will take Σ through the sequence of states
 - $\hat{T}(s_0, \pi) = \langle s_1, \dots, s_n \rangle$
 - then return success
- Note that Σ is unlikely to be a perfect model of the actor's environment
 - Later we'll discuss some things that can go wrong
- To test whether π has achieved a desired goal S_g
 - add S_g as a third argument
 - before line 2, insert this:
 - if** $s \notin S_g$ **then return** failure

Representation

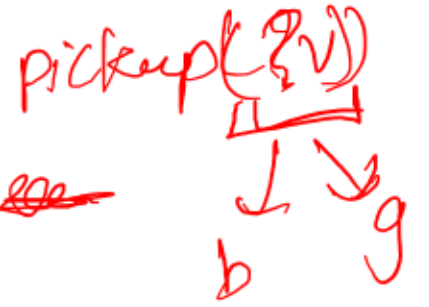
- We write $\text{Run-Plan}(\Sigma, \pi)$
 - But what Run-Plan really needs is data structures that represent Σ and π
- If S and A are small enough
 - Give each state and action a name
 - For each s and a , store $T(s,a)$ in a lookup table
- In larger domains, don't represent all states explicitly, so we need:
 - A language for describing properties of states
 - A language for describing how each action changes those properties
 - A procedure: start with an initial state, use actions to produce other states



Kinds of Representations

pickup ~~b~~

pickup ~~g~~



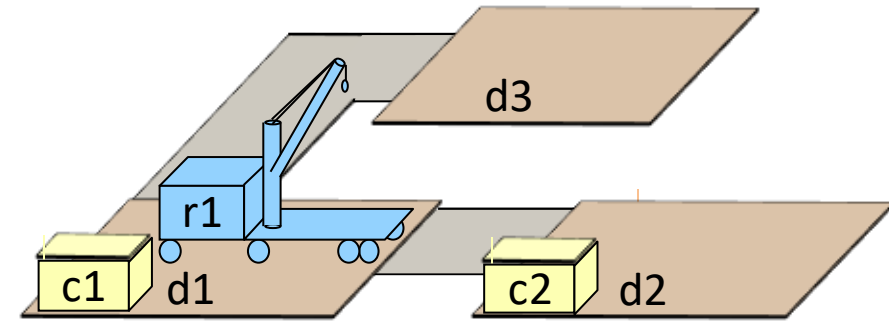
- Domain-specific representation:
 - tailor-made for a specific environment
- State: arbitrary data structure
- Action: (header, preconditions, effects, cost)
 - header: name and parameter list
 - Get actions by instantiating the parameters
 - preconditions:
 - Computational tests to predict whether an action can be performed
 - Should be necessary/sufficient for the action to run without error
 - effects:
 - Procedures that modify the current state
 - cost: procedure that returns a number
 - Can be omitted, default is $\text{cost} \equiv 1$
- Advantage: can use whatever works best for that particular domain
- Disadvantage: for each new domain, need new representation, new algorithms
- Alternative: *domain-independent* representation
 - A “standard format” that can be used for many different planning domains
 - May limit representational capability, but will allow more general compute approaches
 - Domain-independent algorithms that work for anything in this format
 - We’ll use a *state-variable* representation ...

Kinds of Representations

- Advantage of domain-specific representation:
 - use whatever works best for that particular domain
- Disadvantage:
 - for each new domain, need new representation and deliberation algorithms
- Alternative: *domain-independent* representation
 - Try to create a “standard format” that can be used for many different planning domains
 - Deliberation algorithms that work for anything in this format
- *State-variable* representation
 - Simple formats for describing states and actions
 - Limited representational capability
 - But easy to compute, easy to reason about
 - Domain-independent search algorithms and heuristic functions that can be used in all state-variable planning problems

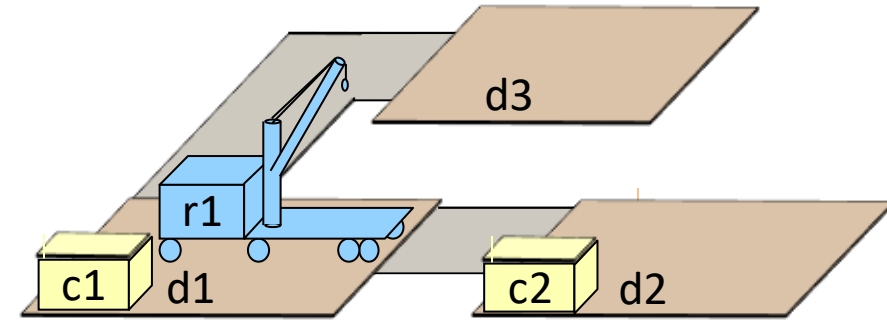
State-Variable Representation

d_1, d_2, d_3 — docks
 c_1, c_2 — containers
 r_1 — robot



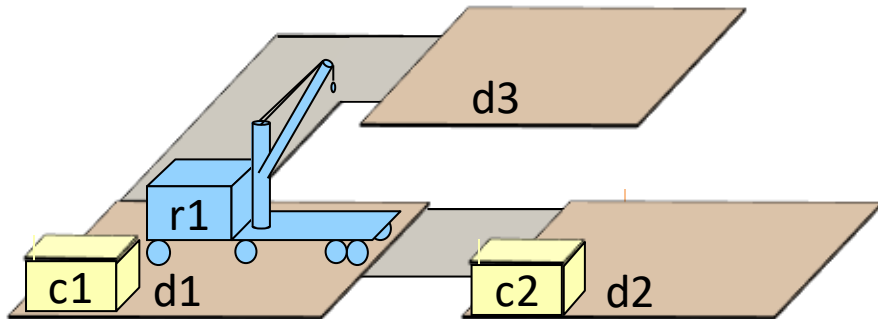
State-Variable Representation

- *Objects* = {names of objects in the environment}
- Organized into a *typed ontology*
 - sets of object types
- *Objects* = *Robots* $\cup$ *Containers* $\cup$ *Locs* $\cup$ {nil}
 - *Robots* = {r1}
 - *Containers* = {c1, c2}
 - *Locs* = {d1, d2, d3}
- *Objects* only needs to include objects that matter at the current level of abstraction
- Can omit lots of details
 - physical characteristics of robots, containers, loading docks, roads, ...
- Objects have two kinds of properties
 - *rigid* and *varying*



Rigid/Static Properties

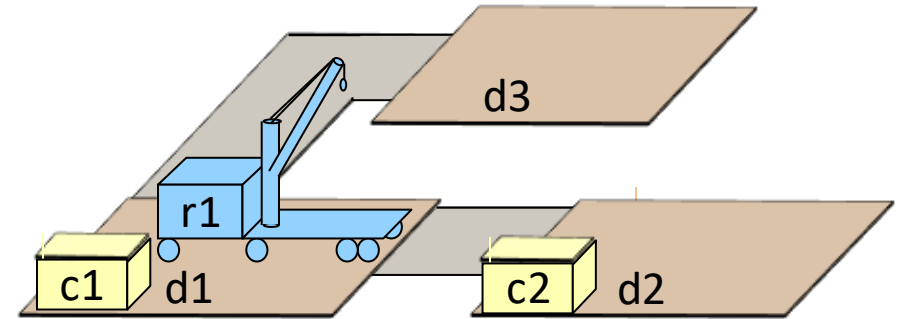
- *Rigid*: stays the same in every state
 - Can be described as a mathematical relation
 $\text{adjacent} = \{(d1,d2), (d2,d1), (d1,d3), (d3,d1)\}$
 - Or equivalently, a set of *ground atoms*
 $\text{adjacent}(d1,d2), \text{adjacent}(d2,d1),$
 $\text{adjacent}(d1,d3), \text{adjacent}(d3,d1)$
 - I'll use the two notations interchangeably



- Terminology from first-order logic:
 - *atom* $\equiv$ *atomic formula* $\equiv$ *positive literal*
 $\equiv$ predicate symbol with list of arguments
 - e.g., $\text{adjacent}(x,d2)$, where x is unbound
 - *negative literal* $\equiv$ *negated atom* $\equiv$ atom with a negation sign in front of it
 - e.g., $\neg \text{adjacent}(x,d2)$
 - an atom is *ground* (or *fully instantiated*) if it contains no variable symbols, *lifted* if it contains all variable symbols, *partially instantiated* if it contains some variables
 - e.g., $\text{adjacent}(d1,d2)$ is ground
 - e.g., $\text{adjacent}(x,d2)$ is partially ground
 - e.g., $\text{adjacent}(x,y)$ is lifted
 - *ground instance* of any expression: replace every variable with a value in its range
 - e.g., $\text{adjacent}(d1,d2)$ is a ground instance of both $\text{adjacent}(x,d2)$ and $\text{adjacent}(x,y)$

Varying Properties

- Varying property (or *fluent*):
 - a property that may differ in different states
- Represent it using a *state variable*
 - a term that we can assign a value to
 - e.g., $\text{loc}(r1)$
- Let $X = \{\text{all state variables in the environment}\}$
e.g., $X = \{\text{loc}(r1), \text{loc}(c1), \text{loc}(c2), \text{cargo}(r1)\}$
- Each state variable $x \in X$ has a *range*
 $= \{\text{all values that can be assigned to } x\}$
 - $\text{Range}(\text{loc}(r1)) = \text{Locs}$
 - $\text{Range}(\text{loc}(c1)) = \text{Range}(\text{loc}(c2)) = \text{Robots} \cup \text{Locs}$
 - $\text{Range}(\text{cargo}(r1)) = \text{Containers} \cup \{\text{nil}\}$
- To abbreviate the “range” notation often we just say things like
 - $\text{loc}(r1) \in \text{Locs}$
 - $\text{loc}(c1), \text{loc}(c2) \in \text{Robots} \cup \text{Locs}$

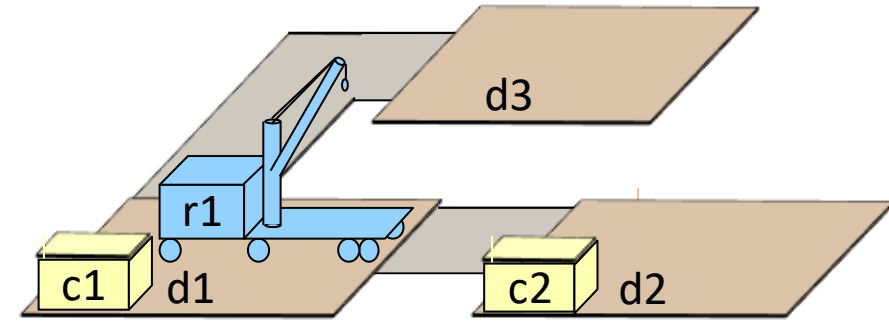


Instead of “domain”,
to avoid confusion
with planning domains

States as Functions

- Represent each state s as a function that assigns values to state variables
 - For each state variable x , $s(x)$ is one x 's possible values

$$\begin{aligned}s_1(\text{loc}(r1)) &= d1, & s_1(\text{cargo}(r1)) &= \text{nil}, \\ s_1(\text{loc}(c1)) &= d1, & s_1(\text{loc}(c2)) &= d2\end{aligned}$$



- Mathematically, a function is a set of ordered pairs
$$s_1 = \{(\text{loc}(r1), d1), (\text{cargo}(r1), \text{nil}), (\text{loc}(c1), d1), (\text{loc}(c2), d2)\}$$
- Equivalently, write it as a set of *ground positive literals* (or *ground atoms*):

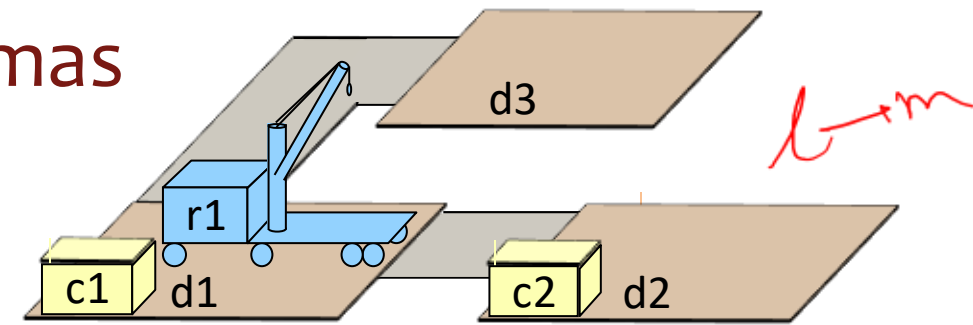
$$s_1 = \{\text{loc}(r1)=d1, \text{cargo}(r1)=\text{nil}, \text{loc}(c1)=d1, \text{loc}(c2)=d2\}$$

- Here, '=' is a predicate symbol
- Assignment will be indicated with '←'

Suppose $r1$ can move between locations. What does the state need to include to represent that $\text{move}(r1, d1, d2)$ is valid while $\text{move}(r1, d2, d3)$ is invalid?

(short answer)

Action Templates or Action Schemas



- Action *template* or *schema*: a parameterized set of actions

$\alpha = (\text{head}, \text{pre}, \text{eff}, \text{cost})$

- head: *name, parameters*
- pre: *precondition literals*
- eff: *effect literals*
- cost: *a number* (optional, default is 1)

- e.g.,

- head = $\text{take}(r, l, c)$
- pre = $\{\text{cargo}(r) = \text{nil}, \text{loc}(r) = l, \text{loc}(c) = l\}$
- eff = $\{\text{cargo}(r) = c, \text{loc}(c) = r\}$

- Each parameter has a range of possible values.

- $\text{Range}(r) = \text{Robots} = \{r1\}$
- $\text{Range}(l) = \text{Locs} = \{d1, d2, d3\}$
- $\text{Range}(l) = \text{Range}(m) = \text{Locs} = \{d1, d2, d3\}$
- $\text{Range}(c) = \text{Containers} = \{c1, c2\}$

- But we'll usually write it more like pseudocode

the *target*
of the
assignment

```

move(r, l, m)
pre: loc(r)=l, adjacent(l, m)
eff: loc(r) ← m

take(r, l, c)
pre: cargo(r)=nil, loc(r)=l,
    loc(c)=l
eff: cargo(r) ← c, loc(c) ← r

put(r, l, c)
pre: loc(r)=l, loc(c)=r
eff: cargo(r) ← nil, loc(c) ← l

r ∈ Robots = {r1}
l, m ∈ Locs = {d1, d2, d3}
c ∈ Containers = {c1, c2}
    
```

Planning Domain Definition Language [PDDL]

C: action move (?x ?y ?z)

pre: robot(?x) ∧
dock(?y) ∧ dock(?z)

eff: ~~loc~~
and(loc(?x ?z)
not(loc(?x ?y)))

action move(?x - robot
 ?y - dock
 ?z - dock)

adjacent(?x ?y)

and (robot(?x)
dock(?y)
dock(?z)
adjacent(?y ?z)
loc(?x ?y))

(: domain cargo-domain

(: predicates

~~adjacent (?:x, ?y)~~

loc (?:x, ?y)

cargo (?:x)

robot (?:x)

)
(: types cargo robot dock)

adjacent (?:x - dock
?:y - dock)

Generating Heuristics

Heuristic Estimates

- Easier in some problems than others!
 - Route finding problem has a natural metric (straight line distance)
 - Most state spaces are not so fortunate
- Online shopping agent
 - State space...
 - Metric?
- Household robot that enters a dark room
 - "Distance" between a dark room and the same room with the light on

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

- Can you think of an admissible heuristic?

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

- Can you think of an admissible heuristic?
 - Number of misplaced tiles (6)
 - Total Manhattan distance ($4 + 0 + 3 + 3 + 1 + 0 + 2 + 1 = 14$)

Methodology for Generating Heuristics

- Create a **relaxed version** of the problem
 - Abstract, or approximate and **easier**
- **Relaxed problems** allow shortcuts
 - Optimal solution cost in relaxed $\leq$ optimal solution cost in real
- Optimal solution cost in the relaxed problem constitutes a heuristic
- But, if the relaxed problem is not easy, computing its optimal can be a challenge!

Where A* is Used in Practice...

- Video game route finding problems
- Robot path planning
- Natural Language Processing
- Speech recognition

Search Algorithms: Summary

- Use **successor** functions to formulate large search problems
- General notion of tree search and graph search
 - Fringe,
 - Node expansion strategies
- Informed vs uninformed search
 - If you can get a cost-to-goal estimate, use it as a heuristic
 - Admissible heuristics are good for guaranteed optimality
- Read further to learn about using inadmissible heuristics
 - (What would A^* do?)
- To see some of these algorithms in action: <https://movingai.com/SAS/>